THE UNIVERSITY OF ALBERTA

FURTHER STUDIES IN THE STRESS STRAIN RELATIONSHIPS OF LIGHTWEIGHT CONCRETE

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
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BY

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ABSTRACT

Concrete cylinders were poured from an expanded shale aggregate and normal sand and gravel aggregate to determine the effect on the modulus of elasticity by the substitution of heavyweight sand for lightweight fines. The total volume of mortar (sand, cement and water) was kept constant while the strength and amount of heavyweight sand were varied.

The tests show that the modulus of elasticity of lightweight concrete is approximately 55% of that of heavyweight concrete while for lightweight concrete using 100% heavyweight sand it is approximately 75%. For intermediate values of heavyweight sand a straight line relationship exists between the percent heavyweight sand substituted (by weight) and the modulus of elasticity.

The increase in the modulus of elasticity by substitution of heavyweight sand for lightweight fines is in proportion to the increase in unit weight of the resulting concrete.

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INTRODUCTION

The use of lightweight concrete in structures has become more and more common, therefore it is important that the stress strain relationship of this material is properly understood.

Simmonds [3)] found that the modulus of elasticity for lightweight concrete using an expanded shale aggregate is approximately 55% of the modulus of elasticity for sand and gravel concrete. This is confirmed by other authors listed in the bibliography of this report.

Since the lower modulus of elasticity of lightweight concrete would make it impossible to develop full concrete flexural strength in a reinforced concrete member, lightweight concrete is at a disadvantage compared to normal sand and gravel concrete. It has been felt however that by the addition of small amounts of heavyweight sand (not enough to increase the unit weight significantly) the modulus of elasticity could be raised considerably and thereby more of the concrete flexural strength could be developed.

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3) S. H. Simmonds - The Stress - Strain Relationship for Lightweight Concrete

Masters Thesis - University of Alberta - 1956

This investigation is an extension of the work done by Simmonds, its object being an evaluation of the increase in the modulus of elasticity by the substitution of heavyweight sand for lightweight fines in lightweight concrete.

CHAPTER 1

SCOPE OF TESTING PROGRAM

The purpose of this investigation is to show the effects of the substitution of heavyweight sand for lightweight fines on the modulus of elasticity of concrete.

Mixes of concrete were poured from lightweight aggregates with 0%, 4%, 7.4%, 10.7%, 25%, 50% and 100% heavyweight sand (by weight) substituted for lightweight fines. For comparative purposes a normal sand and gravel concrete was included in the investigation. Since it was felt that a small amount of sand might raise the modulus of elasticity considerably, there were more mixes of the smaller and what was thought more practical percentages of sand.

The concrete 28 day compressive strengths were designed for 2,500, 3,000, 3,500, 4,000 and 5,000 p.s.i. for each combination of aggregates.

Nine 6" by 12" cylinders were poured from each mix and were tested 3 at 7 days, 3 at 28 days and 3 at 42 days age.

Since only the effect of heavyweight sand on the modulus of elasticity was to be studied all mixes were designed to have a constant mortar volume.

CHAPTER 2

NOTATION

The following notation was used throughout this investigation:

For designed compressive strength at 28 days:

25	-	2500 p.s.i.
30	-	3000 p.s.i.
35	-	3500 p.s.i.
40	-	4000 p.s.i.
50	-	5000 p.s.i.

For combinations of aggregates used in the mix:

H	-	sand and gravel aggregates
OH	-	all lightweight aggregates
4H	-	lightweight aggregates with a 4% by weight heavyweight sand substitution
7.4H	-	lightweight aggregates with a 7.4% by weight heavyweight sand substitution
10.7H	-	lightweight aggregates with a 10.7% by weight heavyweight sand substitution
25H	-	lightweight aggregates with a 25% by weight heavyweight sand substitution
50H	-	lightweight aggregates with a 50% by weight heavyweight sand substitution
100H	-	lightweight aggregates with a 100% by weight heavyweight sand substitution

For the age of the specimens:

A	-	7 days
B	-	28 days
C	-	42 days

For example: The notation 30-10.7H B indicates a 3000 p.s.i. designed concrete made up of lightweight aggregates with a 10.7% by weight substitution of heavyweight sand, and tested at 28 days age.

Throughtout this report the term "all lightweight" concrete is used to describe that concrete which is made up entirely of lightweight aggregates. The term f'_c is generally used to represent the 28 day compressive strength of concrete, however in this report it is used to indicate the compressive strength of the concrete at the particular age when tested . The term "heavyweight sand" is used to describe the fines from sand and gravel aggregates.

CHAPTER 3

MIX: - MATERIALS, PROPORTIONS AND TECHNIQUES

MATERIALS

The aggregates used in this investigation were:

- An expanded shale aggregate commercially known as "Herculite" manufactured by Consolidated Concrete Industries Ltd. in Calgary.
- A sand and gravel aggregate produced by O. K. Construction Ltd. at their Wabamun pit.

Standard Type I portland cement (all from one batch) was used.

Water was obtained directly from the City of Edmonton water mains.

A complete description of the physical properties of the aggregates will not be given in this report. However, sieve analysis on a weight basis of the materials are presented to give an indication of the relative gradations of the aggregates.

The lightweight aggregate was divided into three size fractions: fines 3/16" to pan, medium 3/8" to 3/16", and coarse 3/4" to 3/8" The sand and gravel aggregate was divided into two size fractions: 1" to 1/4" and 1/4" to pan.

TABLE 1

SIEVE ANALYSIS

LIGHTWEIGHT AGGREGATE

COARSE

Sieve Size	% Retained
3/4	0.3
1/2	11.3
3/8	66.0
#4	17.0
Pan	5.4

MEDIUM

Sieve Size	% Retained
3/8	1.1
#4	50.0
#8	41.1
Pan	7.8

FINE

Sieve Size	% Retained	Cum. % Ret.
4	0.8	0.8
8	20.0	20.8
16	15.8	36.6
30	22.1	58.7
50	16.2	74.9
100	14.1	89.0
200	8.8	97.8
Pan	2.2	100.0

Finess Modulus = 2.81

TABLE 2

SIEVE ANALYSIS

HEAVYWEIGHT AGGREGATES

COARSE

Sieve Size	% Retained
1	2.5
3/4	31.1
3/8	57.2
#4	7.8
Pan	3.9

FINE

Sieve Size	% Retained	cum. % Ret.
#4	10.6	
#8	11.5	22.1
#16	14.9	37.0
#30	20.1	57.1
#50	30.9	88.0
#100	8.8	96.8
#200	1.4	98.2
Pan	1.8	100.0

Finess Modulus = 3.01

All aggregates were combined for the concrete by the size fractions in which they were obtained. That is no rescreening was employed to obtain an ideal or equally graded material.

PROPORTIONS

The proportions of the mixes were based on the requirements of normal all lightweight concrete and since a constant volume of mortar was desired, this proportioning was used throughout. By keeping the mortar volume constant the exact influence of the mortar as it changes from lightweight to heavyweight could be determined.

The sand and gravel concrete was proportioned with the same volume of mortar as was used in the lightweight concrete. This resulted in an extremely oversanded mix. Cement requirements for this concrete were determined on the basis of a water cement ratio law.

The cement requirements for the all lightweight concrete were determined on an cement versus strength basis at a constant slump relationship. The same cement contents were also used for the mixes with heavyweight sand substituted. This resulted in strengths higher than was designed. The heavyweight sand was substituted in these mixes on a percent weight basis.

Table 3 shows a summary of the mix proportions used, based on one cubic yard.

MIXING TECHNIQUES

The lightweight aggregate when obtained had a moisture content in excess of 10%. It was therefore felt that "pre-soaking" the aggregates would be of no advantage. The materials were batched directly from the storage bins, care being taken not to use any aggregate which had been air dried while in storage.

The concrete was mixed in two cubic foot batches for four minutes in a three cubic foot mixer. In order to get thorough mixing of the lightweight materials the barrel of the mixer was tilted to the verge of discharging (see Photograph 1). This technique was not used in the sand and gravel concrete mixes.

The concrete was discharged into a metal pan after completion of the mixing time, where the slump was measured and the concrete placed into 6" x 12" cardboard molds. The cylinders were capped (to prevent surface drying) and stored in air for 24 hours. At the end of this time the cylinders were stripped and cured until testing in the moisture room.

TABLE 3

DESIGN PROPORTIONS BY WEIGHT FOR CONCRETE

BASED ON 1 CU. YARD. ALL WEIGHTS IN LBS.

Mix *	Cement	Water	Fine		Medium	Coarse
25-H	455	325	1570			1510
30-H	503	325	1530			1510
35-H	555	325	1490			1510
40-H	612	325	1440			1510
50-H	732	325	1340			1510
25-OH	500	340	1060		330	440
30-OH	590	340	1000		330	440
35-OH	670	340	955		330	440
40-OH	740	340	915		330	440
50-OH	920	340	805		330	440
			heavy	light		
25-4H	500	340	42.5	1025	330	440
30-4H	590	340	40	970	330	440
35-4H	670	340	38.5	920	330	440
40-4H	740	340	37	880	330	440
50-4H	920	340	32.5	780	330	440

* See Chapter 2 "Notation" for explanation of symbols

This is the theoretical water required for a 3" slump, but does not include water that is absorbed by the aggregate

TABLE 3 (CONT'D.)

Mix	Cement	Water	Fine Heavy	Fine Light	Medium	Coarse
25-7.4H	500	340	80	1000	330	440
30-7.4H	590	340	76	945	330	440
35-7.4H	670	340	72	900	330	440
40-7.4H	740	340	69	860	330	440
50-7.4H	920	340	58	758	330	440
25-10.7H	500	340	116	970	330	440
30-10.7H	590	340	110	918	330	440
35-10.7H	670	340	105	874	330	440
40-10.7H	740	340	100	838	330	440
50-10.7H	920	340	88	735	330	440
25-25H	500	340	284	852	330	440
30-25H	590	340	269	808	330	440
35-25H	670	340	255	765	330	440
40-25H	740	340	244	734	330	440
50-25H	920	340	215	645	330	440
25-50H	500	340	615	615	330	440
30-50H	590	340	582	582	330	440
35-50H	670	340	555	555	330	440
40-50H	740	340	530	530	330	440
50-50H	920	340	465	465	330	440

TABLE 3 (CONT'D.)

Mix	Cement	Water	Fine Heavy	Fine Light	Medium	Coarse
25-100H	500	325	1490	0	330	440
30-100H	590	325	1410	0	330	440
35-100H	670	325	1340	0	330	440
40-100H	740	325	1280	0	330	440
50-100H	920	325	1130	0	330	440

Photograph 1 - Mixer showing Position of Barrel for mixing Lightweight Aggregates

CHAPTER 4

APPARATUS

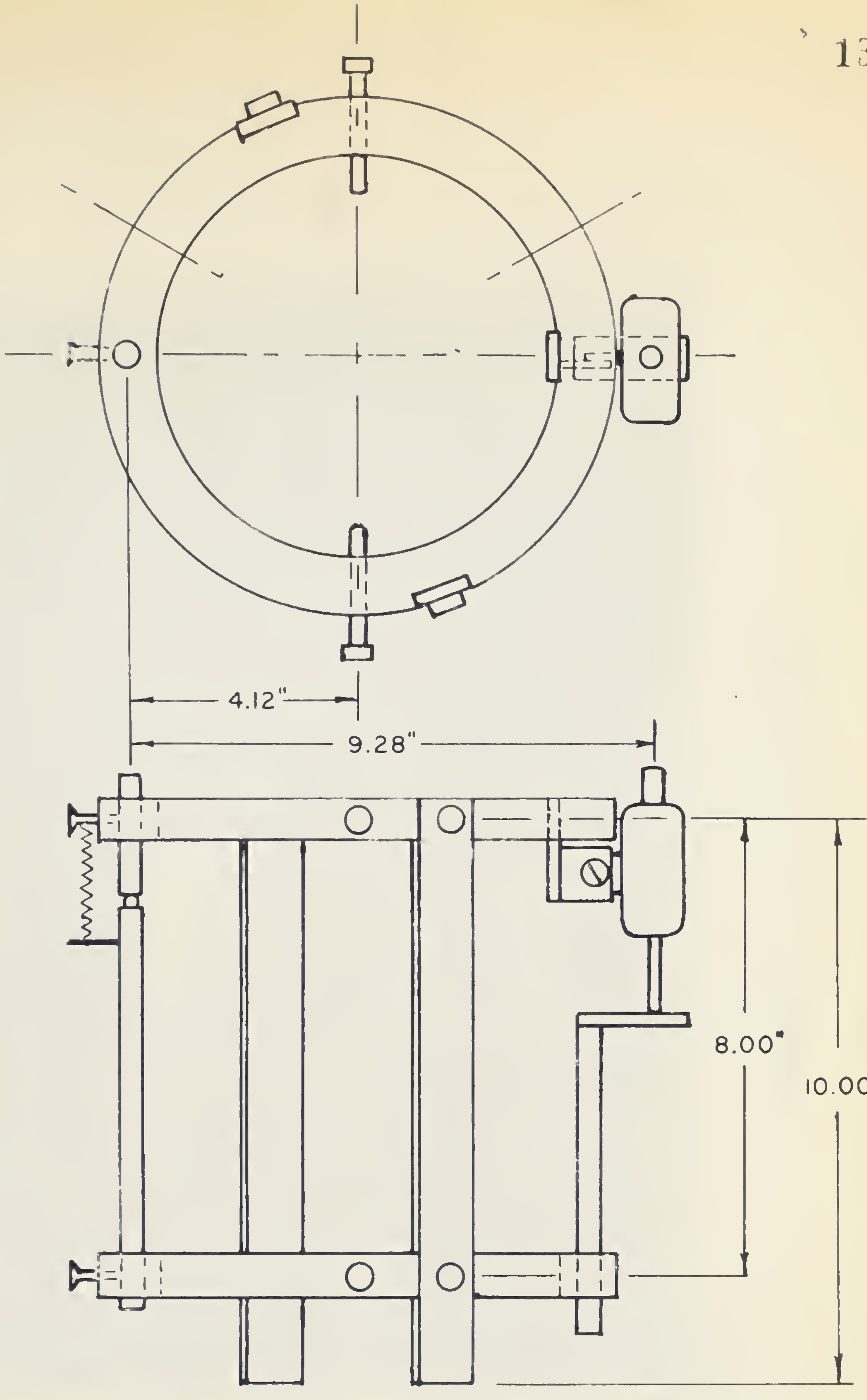
The cylinders were tested in a hydraulic testing machine having a 300,000 pound capacity. Total strains were obtained by the use of an extensometer.

The extensometer consisted of a bottom ring which fastened rigidly to the cylinder by means of three set screws placed 120° apart, and a top ring which fastened to the cylinder by two diametrically opposite set screws which allowed the ring to rotate about the axis of the set screws. The top ring hinged at one side on a rod held rigid to the bottom ring. Diametrically opposite a 10,000th inch "Mercer Dial" was mounted on the top ring and it was seated on a pedestal from the bottom ring (see Figure 1 and Photograph 2). The multiplication factor of the extensometer was 2.25 .

The extensometer was mounted symmetrically on the cylinders using an 8" gauge length (Photograph 3).

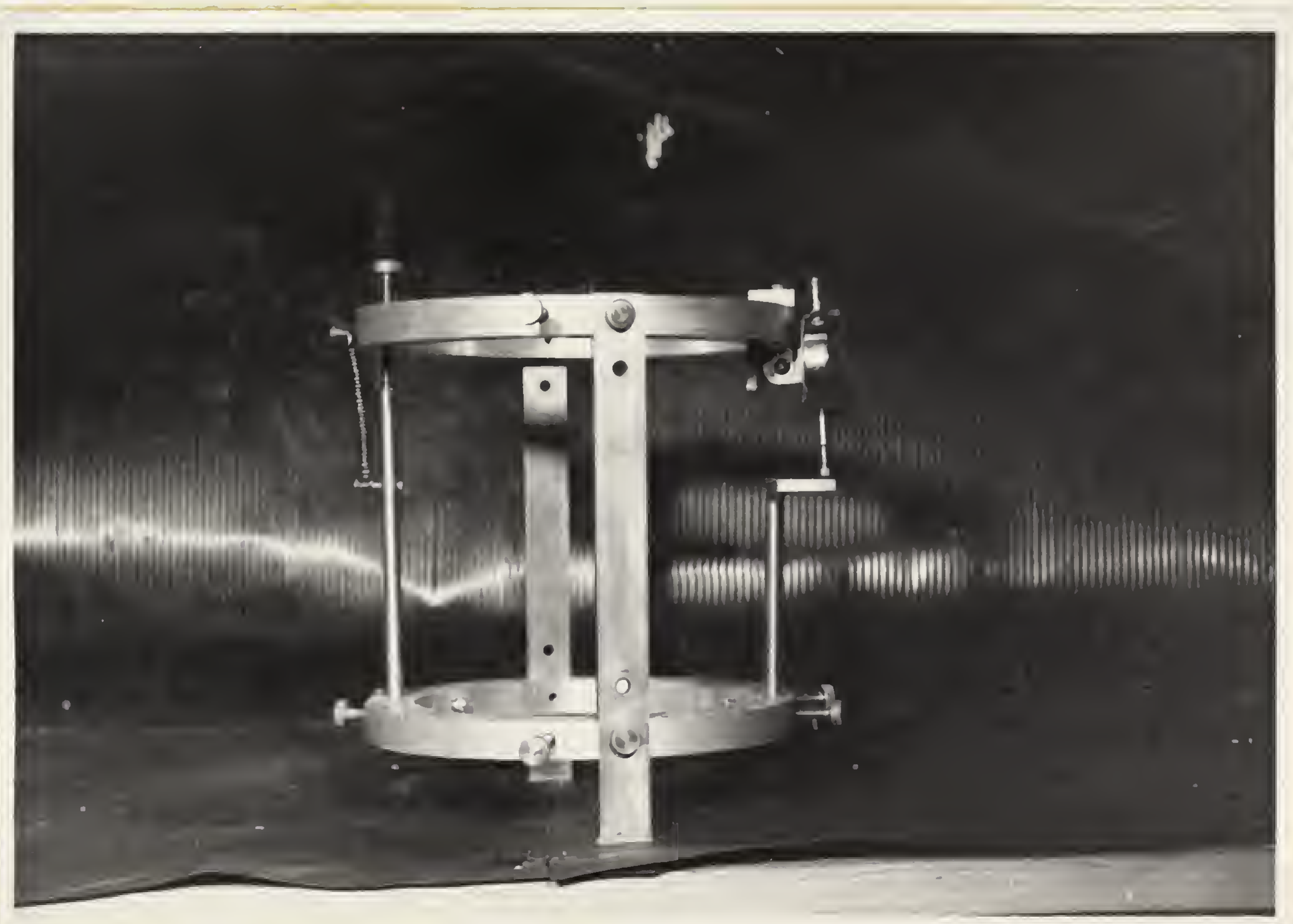
Photograph 4 shows a cylinder and the extensometer in the compression machine for testing.

In general the cylinders were stressed to the ultimate load and then released before complete failure took place. However, two cylinders were failed to show an exposed surface of the concrete. They are shown in Photograph 5.

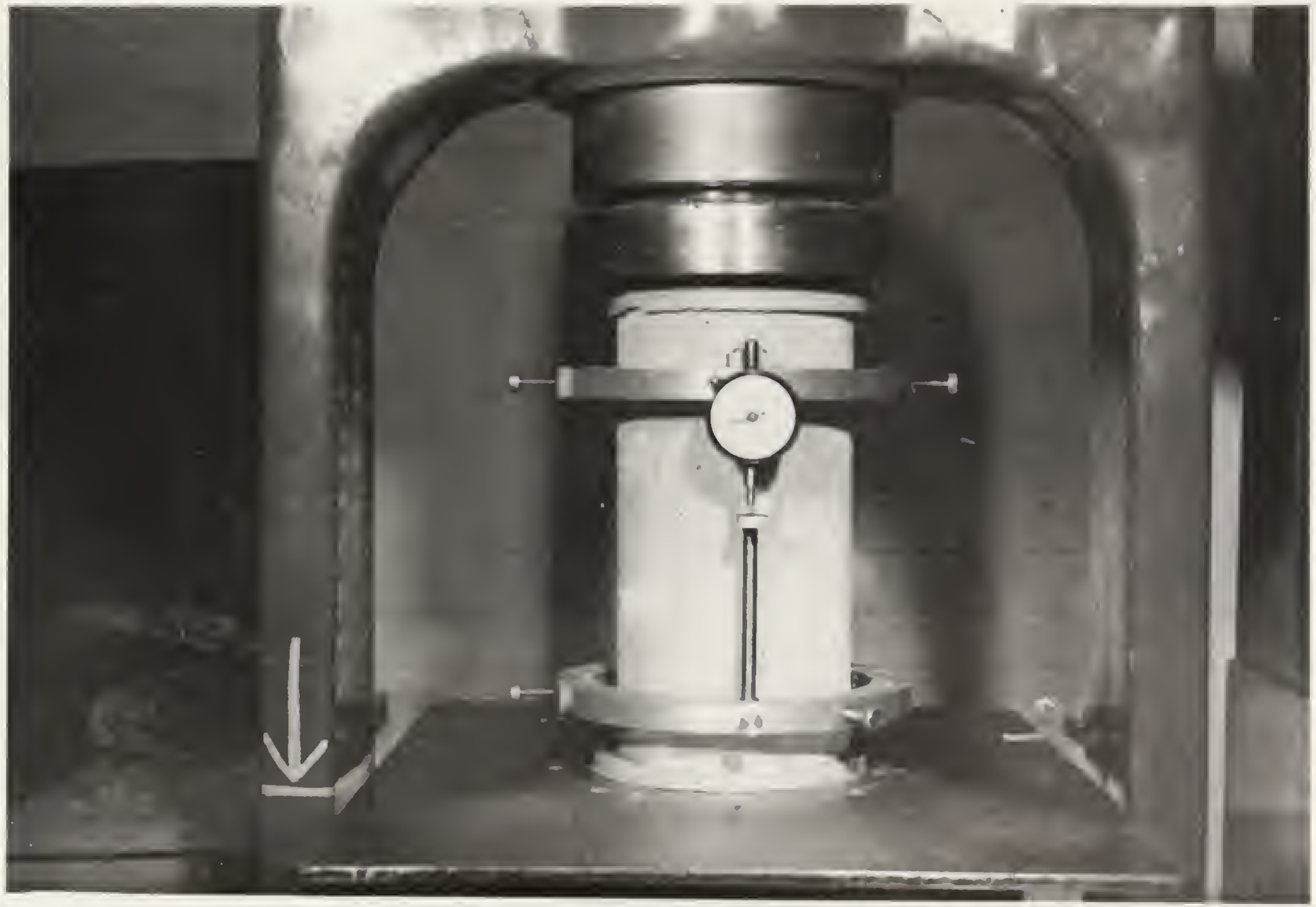


EXTENSOMETER DIMENSIONS

FIGURE I

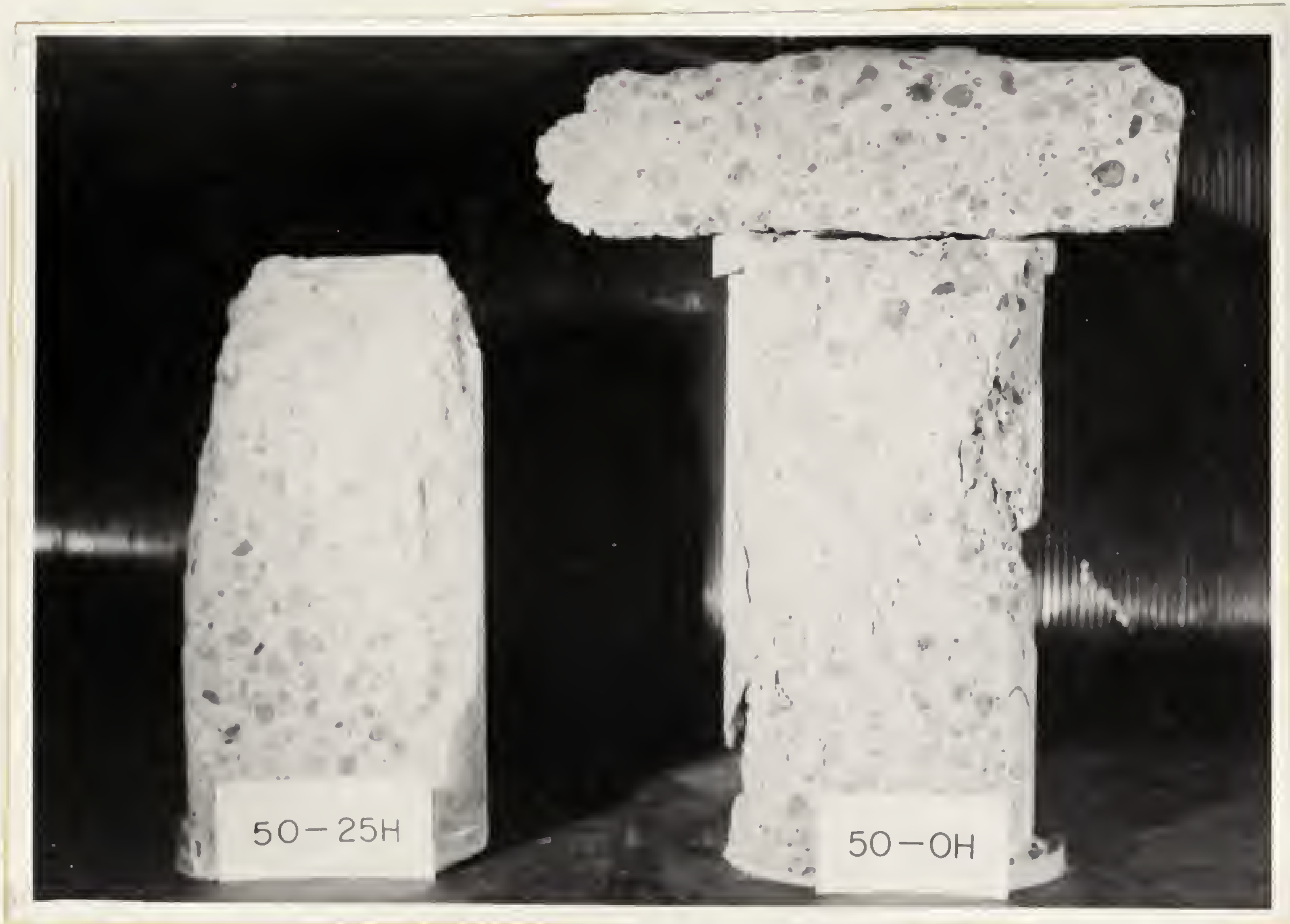


Photograph 2 - The Extensometer



Photograph 3 - Extensometer Mounted on a Cylinder

Photograph 4 - A Cylinder during Testing



Photograph 5 - Exposed View of Concrete

CHAPTER 5

DISCUSSION OF TESTS

The load deformation curves for the 360 cylinders tested were obtained by simultaneous reading of the Mercer Dial and the testing machine while in continuous motion. Since the standard loading rate of 3,000 p.s.i. per minute resulted in a dial speed too fast for accurate reading, a rate of approximately 2,000 p.s.i. per minute was used.

Troxell and Davis [1)] state that a time change from 2 to 20 minutes to test a cylinder to failure has no significant effect on the stress strain curve up to the working stress. Therefore the change of loading rate from 3,000 to 2,000 p.s.i. per minute would be of little consequence to the resulting stress strain curve.

The modulus of elasticity reported herein is based on the slope of the secant passing through the origin and the point on the stress-strain curve corresponding to 45% of the ultimate strength of the concrete cylinder. The value was chosen as it represents the allowable flexural stress by the A.C.I. and National Building Codes. This method seemed preferable to the initial tangent method since it would give a more representative moduli of elasticity and

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1) Troxell and Davis - "Concrete"
McGraw Hill

the latter method involved a certain degree of personal judgement in setting the slope.

All cylinders were tested within 1/2 to 1-1/2 hours after removal from the moist room. The modulus of elasticity will therefore be somewhat higher than for cylinders allowed to dry to a considerable extent (Troxell and Davis [1)]).

The stress strain data sheets and graphs are not included in this report, however copies are on file at the office of the Department of Civil Engineering, University of Alberta. The modulus of elasticity and compressive strengths for each cylinder tested are recorded in table 4. The stress strain curve for a typical cylinder and the method used in computing the modulus of elasticity are shown in Figure 2.

A graph of the modulus of elasticity versus compressive strength for each combination of aggregates is plotted in Figures 3 to 10. In Figure 11 these results are superimposed to aid in comparison.

Since very few tests were run for compressive strengths of less than 2,500 p.s.i. the shape of the curve is unknown in this range. For this reason the curves were not plotted beyond the limits of the testing range.

On all plots the 7, 28 and 42 day results were not seperated because the limited number of tests for each combination of aggregates would not enable any relationship to be developed over a

significant range. The majority of 7 day results fall below the plotted line and in every case points which are appreciably below the line are 7 day results. This indicates that the younger the concrete the lower the modulus of elasticity at any one compressive strength. This would seem to be supported by the pre-stressed concrete industry where larger cambers are experienced in post-tensioned girders stressed at a younger age, all girders having equal compressive strengths. It should be noted however that this observation applies only to 7 day test results and the relationship should not be inferred beyond this range.

The relationship between the modulus of elasticity and the compressive strength for sand and gravel mix used in this investigation is not truly representative of the relationship that one would obtain for a normal sand and gravel concrete. This is due to the fact that the concrete in question was extremely oversanded because of the constant mortar requirements of the investigation. Thus this curve is not typical of the relationship of normal sand and gravel concrete, but it is useful to study the effects of the mortar on the modulus of elasticity. A typical relationship between the modulus of elasticity and the compressive strength for a properly proportioned sand and gravel concrete obtained from Richart and Jensen [2)] is shown in Figure 11 and 16.

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2) F. E. Richart and V. P. Jensen - Tests of Plain and Re-inforced Concrete made with Haydite Aggregates.
Bulletin #237 University of Illinois, Engineering Experimental Stn.

FIGURE 2

A TYPICAL STRESS – STRAIN CURVE
FOR THE 50 -OH – B MIX

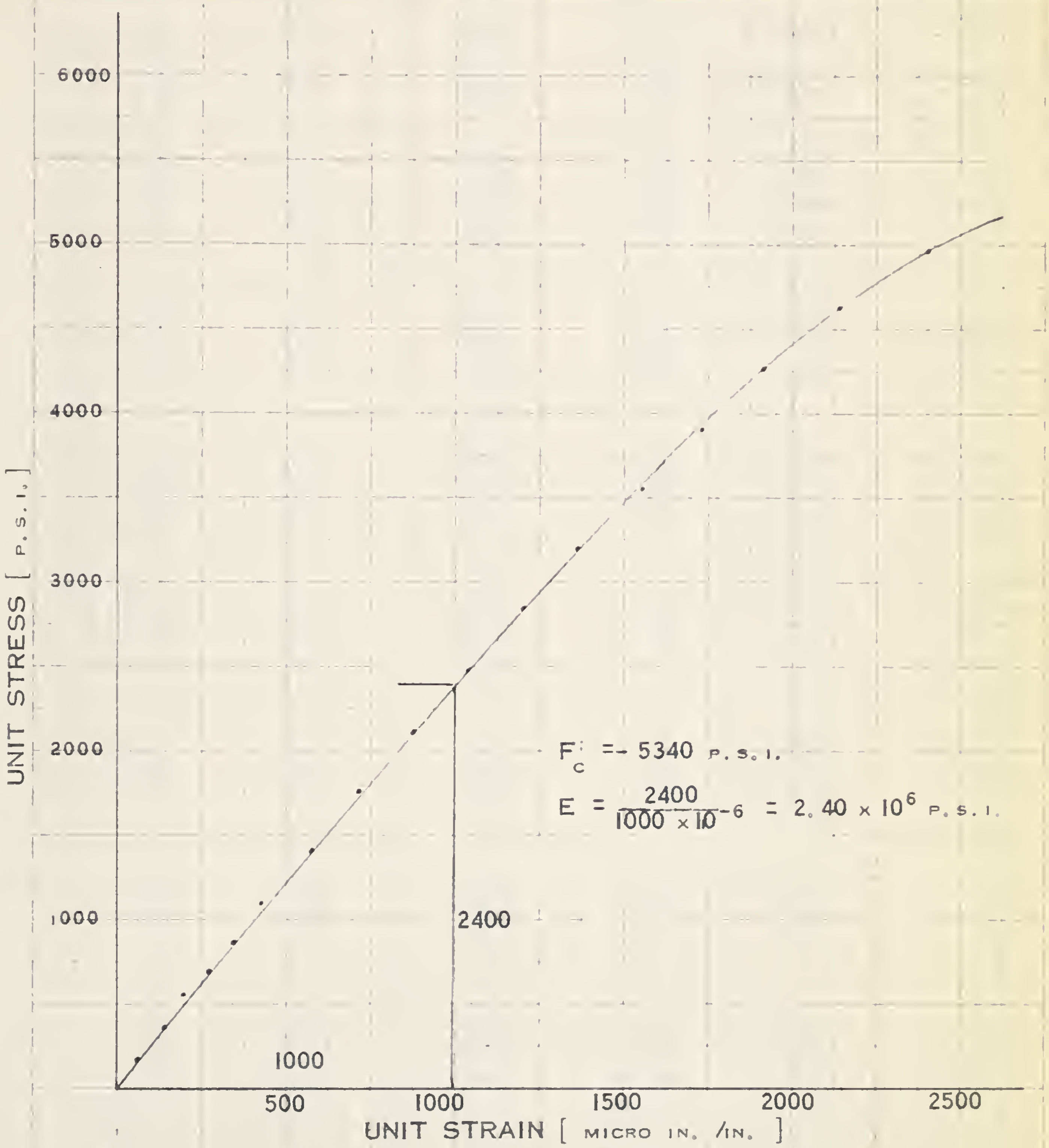


TABLE 4

DATA

Mix		f'_c	$E_c x 10^6$	f'_c	$E_c x 10^6$	f'_c	$E_c x 10^6$
25-H	A	2060	2.53	2100	2.62	2060	2.52
	B	3010	3.11	2870	3.12	-	-
	C	3280	3.31	3280	3.31	3280	3.20
30-H	A	2580	2.64	2660	2.86	2640	2.42
	B	3440	3.32	3600	3.03	3440	3.44
	C	3850	3.44	3826	3.20	3730	3.10
35-H	A	3350	3.12	3480	3.07	3400	3.10
	B	4660	3.80	4230	3.59	4490	3.55
	C	4900	3.80	4850	3.81	4800	3.72
40-H	A	3820	2.97	3840	3.06	3720	3.15
	B	4450	3.64	4770	3.60	4670	3.54
	C	5300	3.70	4870	3.60	5040	3.73
50-H	A	4070	3.22	4180	3.19	4180	3.19
	B	4830	3.55	4550	3.42	4820	3.82
	C	5540	3.78	5050	3.59	5340	3.60
25-100H	A	2730	2.40	2750	2.42	2820	2.46
	B	3860	2.90	3900	2.82	3650	2.82
	C	4190	2.92	4080	3.02	3960	2.92
30-100H	A	3560	2.40	3550	2.44	3510	2.22
	B	4860	3.02	4690	2.94	4560	2.96
	C	5070	2.72	5000	3.11	5080	3.07
35-100H	A	3720	2.60	3720	2.60	3700	2.71
	B	4760	3.06	5030	2.93	4800	3.06
	C	5020	3.00	5180	3.00	5020	3.08
40-100H	A	3600	2.81	4120	2.42	4300	2.76
	B	4830	3.07	5210	3.06	5380	3.11
	C	5250	3.05	5320	3.00	5350	2.74

TABLE 4 (CONT'D.)

Mix		f'_c	$E_c x 10^6$	f'_c	$E_c x 10^6$	f'_c	$E_c x 10^6$
50-100H	A	4820	2.75	4850	2.82	4140	2.85
	B	5930	3.20	5680	3.11	5650	3.18
	C	5510	3.17	5480	3.17	5650	3.15
25-50H	A	2620	1.98	2640	2.02	2570	1.96
	B	3840	2.47	3650	2.33	3660	2.23
	C	4080	2.43	3900	2.50	3960	2.28
30-50H	A	2980	2.02	2800	2.06	2540	2.02
	B	4110	2.40	4080	2.42	4050	2.36
	C	4300	2.38	4400	2.38	4400	2.56
35-50H	A	3400	2.10	3440	2.13	-	-
	B	4660	2.54	4340	2.40	4660	2.52
	C	4810	2.70	4650	2.62	4680	2.56
40-50H	A	3960	2.24	3620	2.14	3600	2.37
	B	4820	2.69	5040	2.65	4960	2.59
	C	4850	2.82	5060	2.80	4930	2.85
50-50H	A	4030	2.02	3990	2.02	4380	2.19
	B	5200	2.58	5240	2.60	5260	2.66
	C	5050	2.84	5450	2.73	5510	2.70
25-25H	A	2750	1.87	2660	1.91	2620	1.95
	B	3880	2.25	3900	2.29	3690	2.22
	C	4140	2.32	4330	2.31	4220	2.28
30-25H	A	2540	1.93	2600	1.99	-	-
	B	3900	2.31	3850	2.29	3980	2.22
	C	3640	2.40	4190	2.35	3960	2.34
35-25H	A	2780	1.91	2760	1.93	2710	1.91
	B	4200	2.28	4280	2.35	3970	2.22
	C	4220	2.44	4500	2.36	4460	2.43

TABLE 4 (CONT'D.)

Mix		f'_c	$E_c x10^6$	f'_c	$E_c x10^6$	f'_c	$E_c x10^6$
40-25H	A	3300	2.06	3320	2.07	3160	2.00
	B	4660	2.54	4330	2.35	4860	2.40
	C	4550	2.43	5130	2.41	4960	2.46
50-25H	A	4030	2.18	4280	2.20	4430	2.18
	B	5350	2.58	5290	2.50	4990	2.41
	C	5120	2.66	5320	2.66	5500	2.62
25-10.7H	A	1840	1.40	1790	1.51	1720	1.45
	B	2720	1.73	2910	2.08	2880	1.78
	C	3050	1.94	3010	1.73	3010	1.97
30-10.7H	A	2460	1.93	2480	1.84	2540	1.99
	B	3580	1.99	3600	2.10	3640	2.07
	C	3920	2.16	3860	2.18	3900	2.15
35-10.7H	A	3280	2.16	3240	2.03	3170	1.97
	B	4370	2.24	4290	2.20	4270	2.28
	C	4910	2.31	4740	2.31	4790	2.35
40-10.7H	A	3230	1.95	3300	1.97	3260	1.87
	B	4360	2.26	4610	2.28	4590	2.28
	C	4950	2.38	5200	2.37	5050	2.43
50-10.7H	A	3870	2.07	3880	2.07	3900	2.08
	B	5330	2.39	5010	2.36	5000	2.37
	C	5400	2.46	5300	2.45	5260	2.51
25-7.4H	A	1880	1.73	1850	1.66	1920	1.70
	B	2700	2.10	3090	2.08	3000	2.08
	C	3190	2.10	3250	2.07	3080	2.10
30-7.4H	A	2560	1.92	2560	1.90	2540	1.94
	B	3480	2.17	3740	2.21	3670	2.17
	C	4080	2.26	4110	2.25	3970	2.25

TABLE 4 (CONT'D.)

Mix		f'_c	$E_c x10^6$	f'_c	$E_c x10^6$	f'_c	$E_c x10^6$
35-7.4H	A	3400	1.91	3280	1.91	3460	1.99
	B	4600	2.23	4600	2.28	4700	2.23
	C	4650	2.32	-	-	5040	2.32
40-7.4H	A	3600	2.02	3740	1.99	3530	1.99
	B	5000	2.33	4860	2.29	4600	2.31
	C	5420	2.44	5020	2.46	5180	2.44
50-7.4H	A	4240	2.03	3980	2.12	4110	2.15
	B	5340	2.53	5110	2.46	5000	2.41
	C	5500	2.54	5380	2.47	5450	2.56
25-4H	A	1980	1.65	2020	1.74	1840	1.67
	B	3010	2.00	3030	2.06	3140	2.02
	C	3330	2.17	3480	2.20	3480	2.20
30-4H	A	2750	1.80	2660	1.81	2730	1.94
	B	4000	2.13	4100	2.16	4100	2.21
	C	4320	2.27	4450	2.30	4350	2.21
35-4H	A	3120	1.93	3170	1.96	3030	1.90
	B	4220	2.18	4410	2.22	4360	2.21
	C	4790	2.34	4540	2.31	4600	2.34
40-4H	A	3260	1.87	3100	1.93	2940	2.89
	B	4300	2.15	4150	2.21	4330	2.18
	C	4880	2.34	4810	2.34	4780	2.34
50-4H	A	4270	2.07	4220	2.09	4190	2.05
	B	5500	2.38	5460	2.45	4900	2.22
	C	5530	2.34	5550	2.34	5600	2.50
25-OH	A	1950	1.72	1860	1.72	1760	1.62
	B	3220	2.04	3330	2.04	3540	2.12
	C	3640	2.14	3580	2.16	3700	2.20

TABLE 4 (CONT'D.)

Mix		f'_c	$E_c x10^6$	f'_c	$E_c x10^6$	f'_c	$E_c x10^6$
30-OH	A	2430	1.74	2400	1.64	2530	1.66
	B	3710	2.05	3760	2.12	3460	2.10
	C	3880	2.28	4300	2.26	4330	2.25
35-OH	A	3500	1.90	3550	1.87	3420	1.86
	B	5090	2.22	5050	2.31	4630	2.29
	C	5240	2.36	5180	2.34	4960	2.38
40-OH	A	3760	1.98	3820	1.98	3720	1.88
	B	4830	2.35	5050	2.29	4890	2.42
	C	5260	2.40	5150	2.40	5350	2.40
50-OH	A	4250	2.06	4080	2.04	4050	2.06
	B	5320	2.40	5370	2.34	5460	2.40
	C	5700	2.57	5500	2.48	5920	2.40

FIGURE 3

MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

for the OH mix

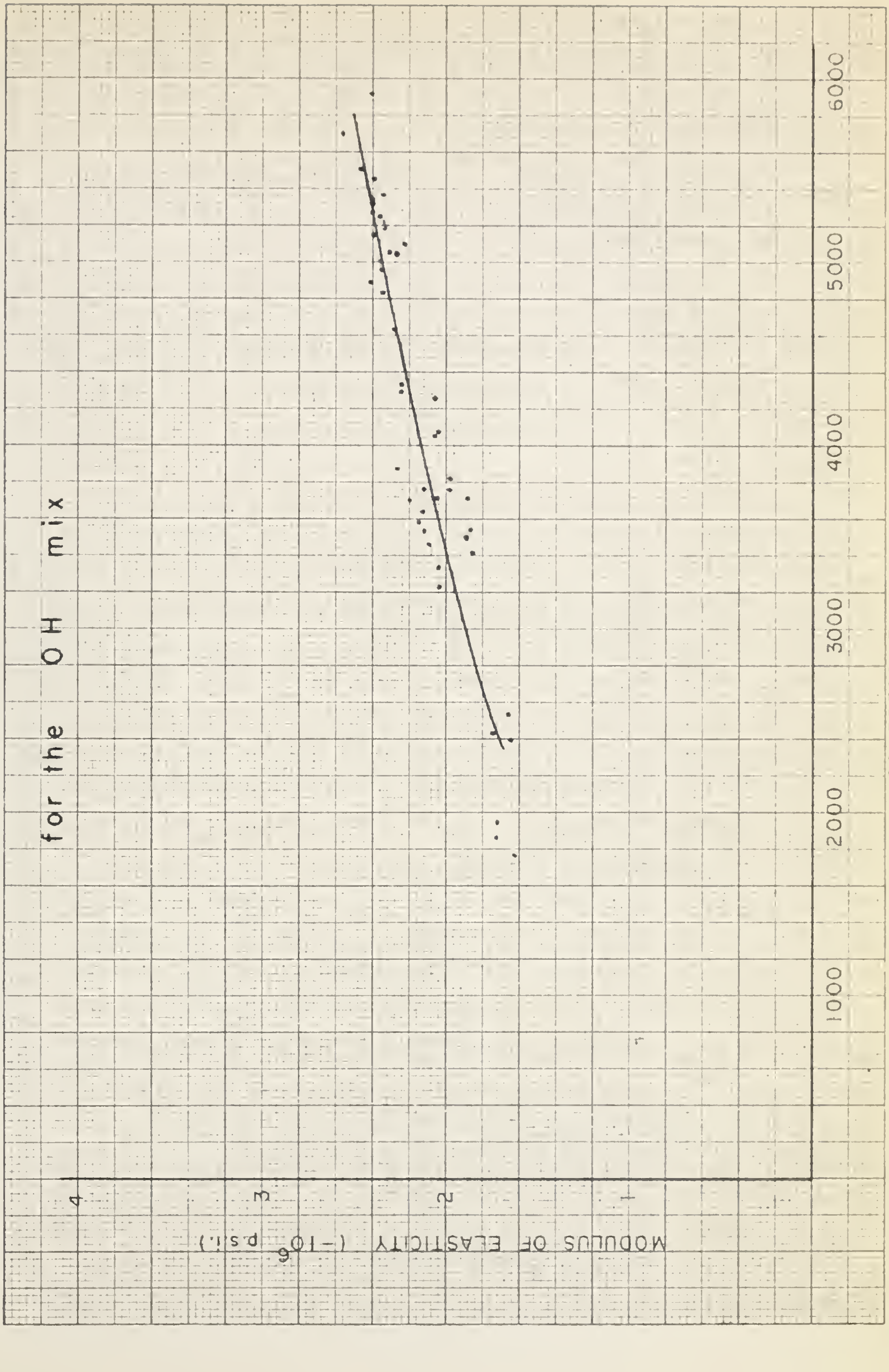


FIGURE 4

MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

for the 4 H mix

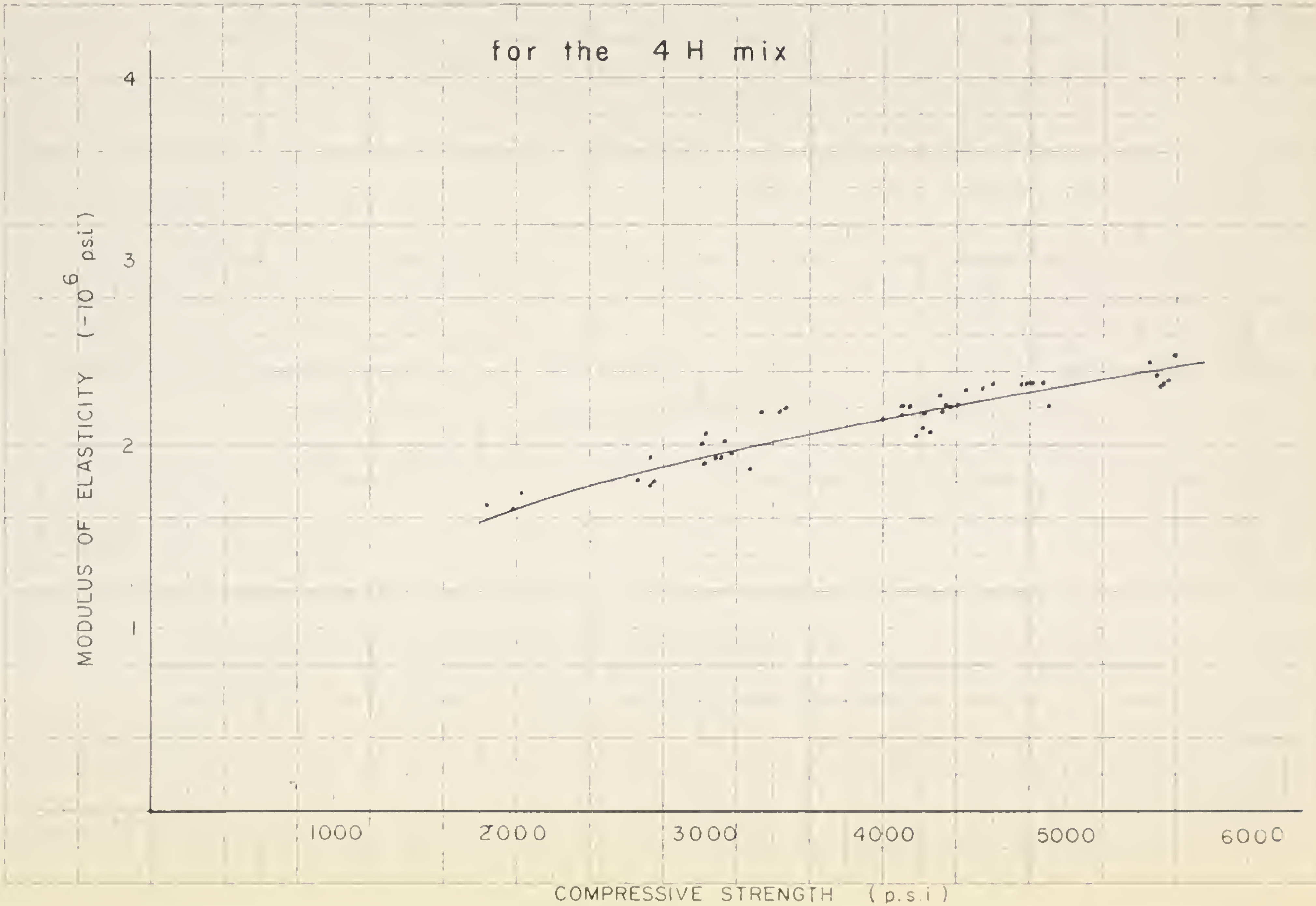


FIGURE 5

MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

for the 7.4 H mix

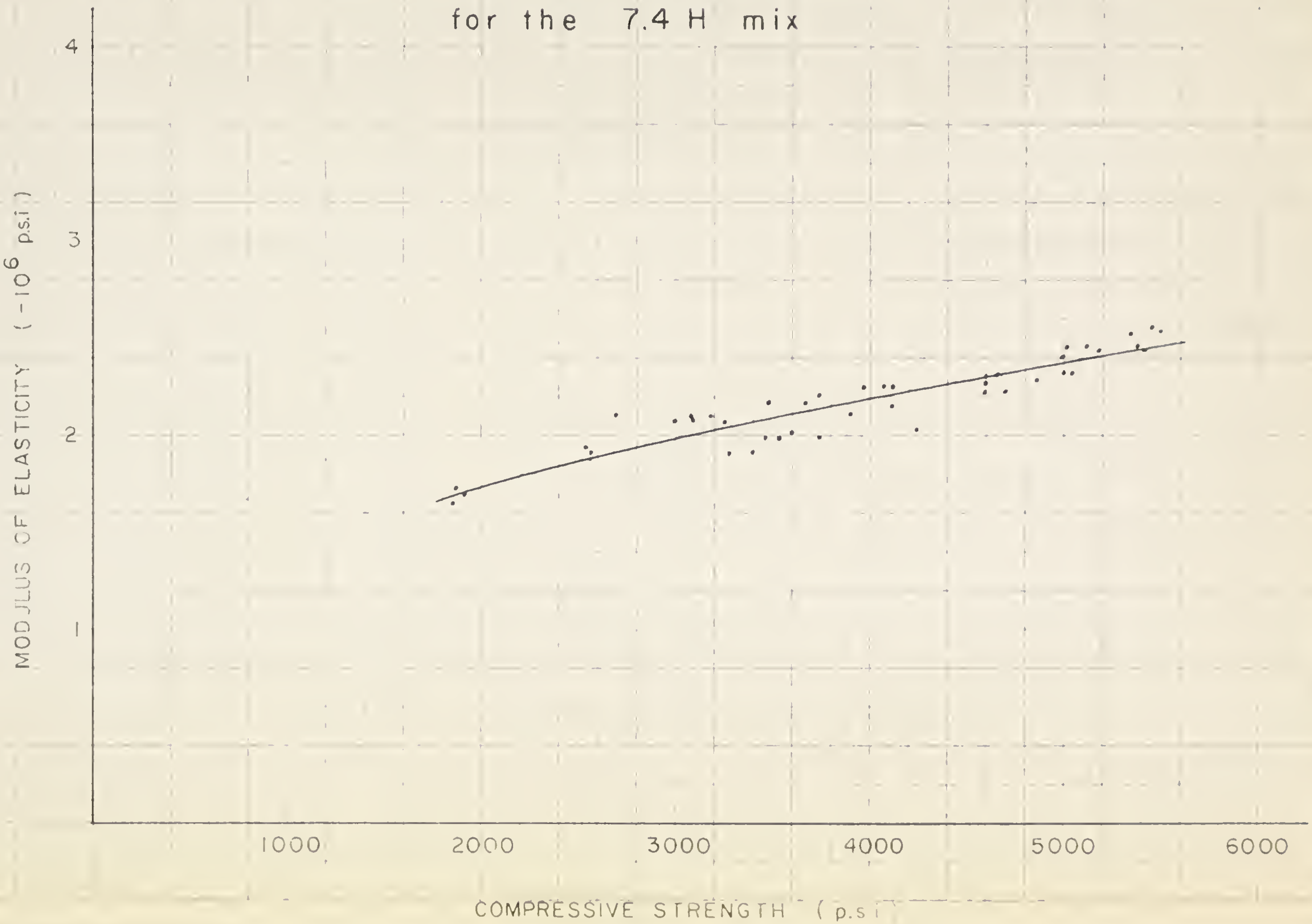


FIGURE 6

MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

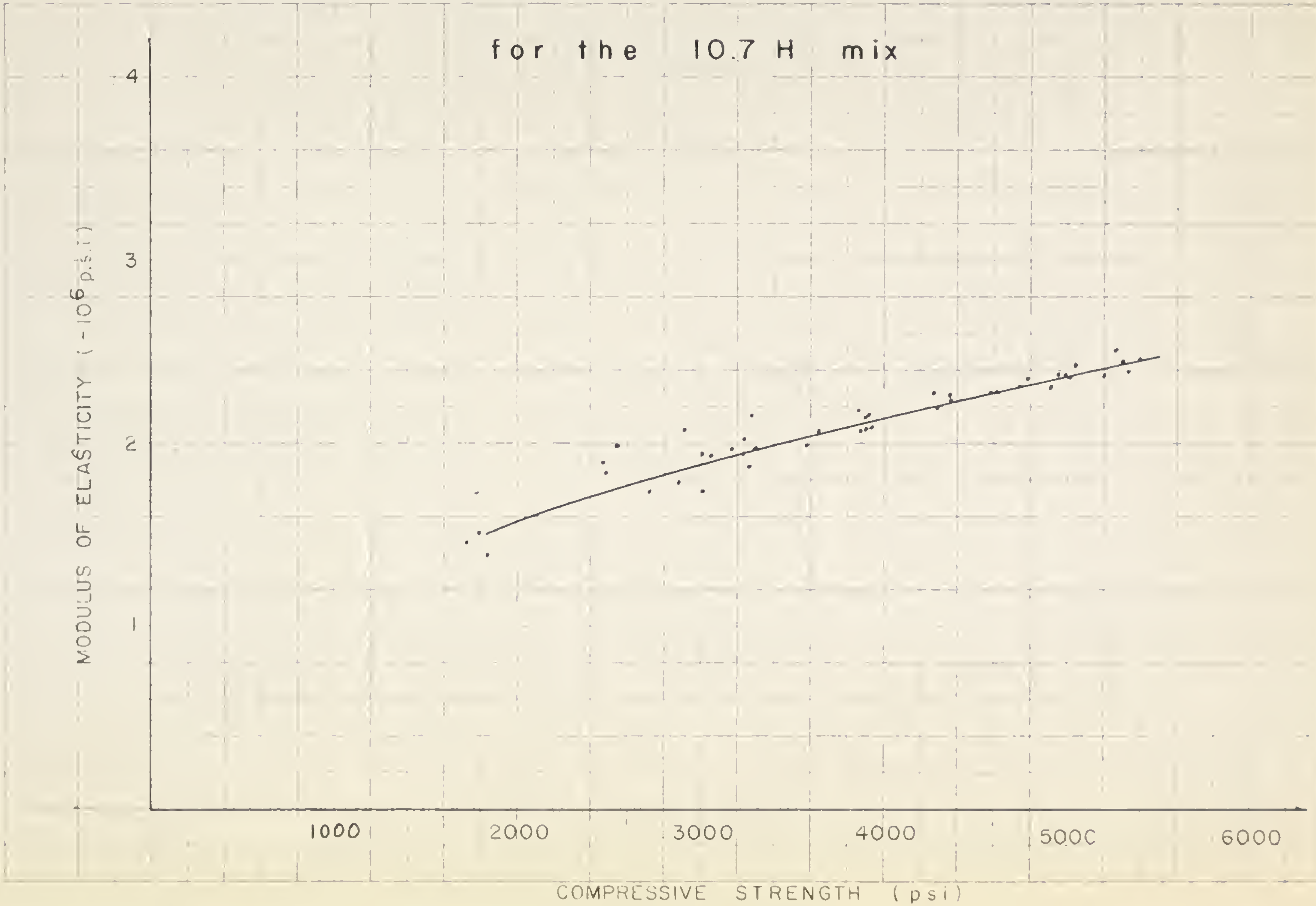


FIGURE 7

MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

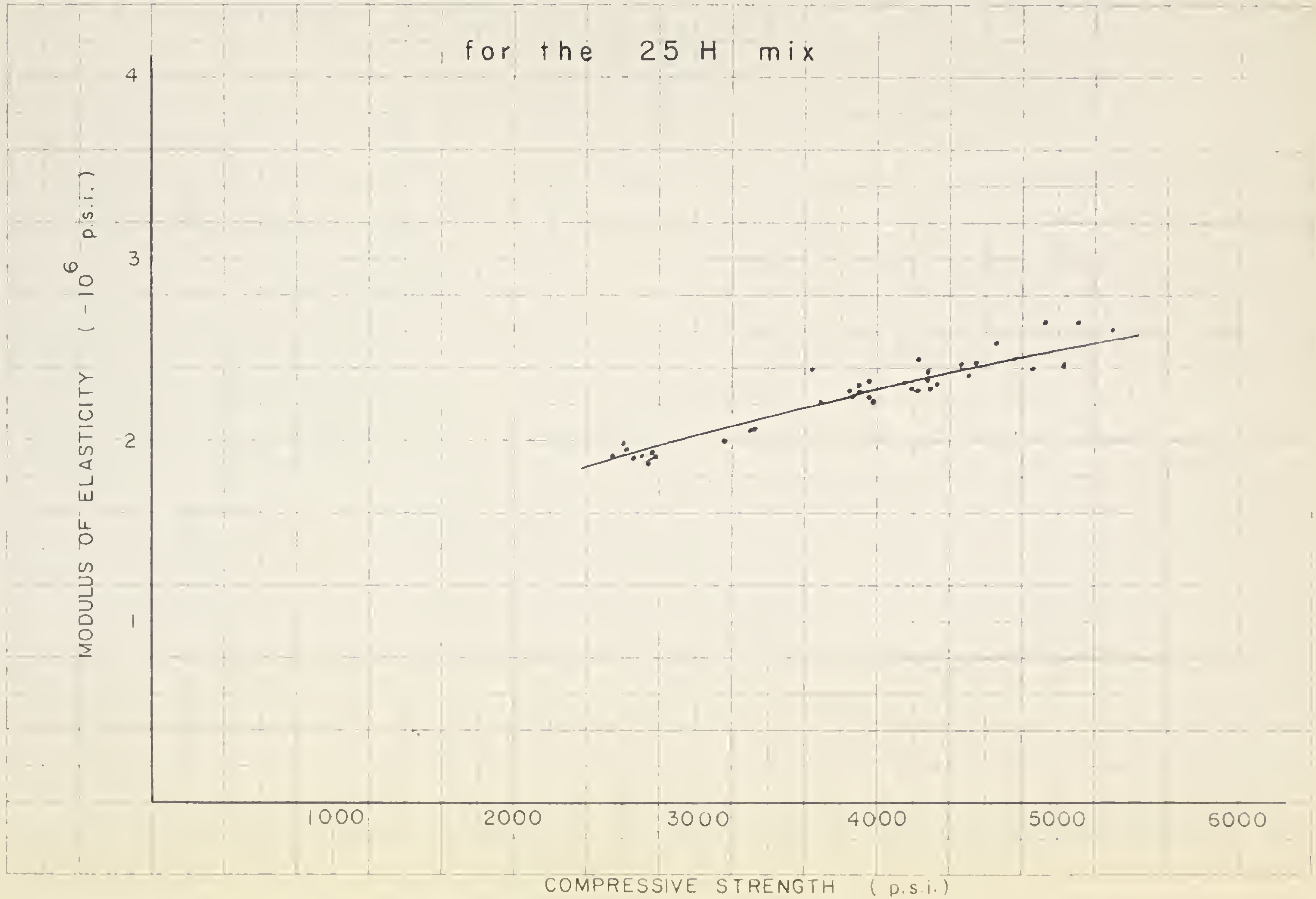


FIGURE 8

MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

for the 50 H mix

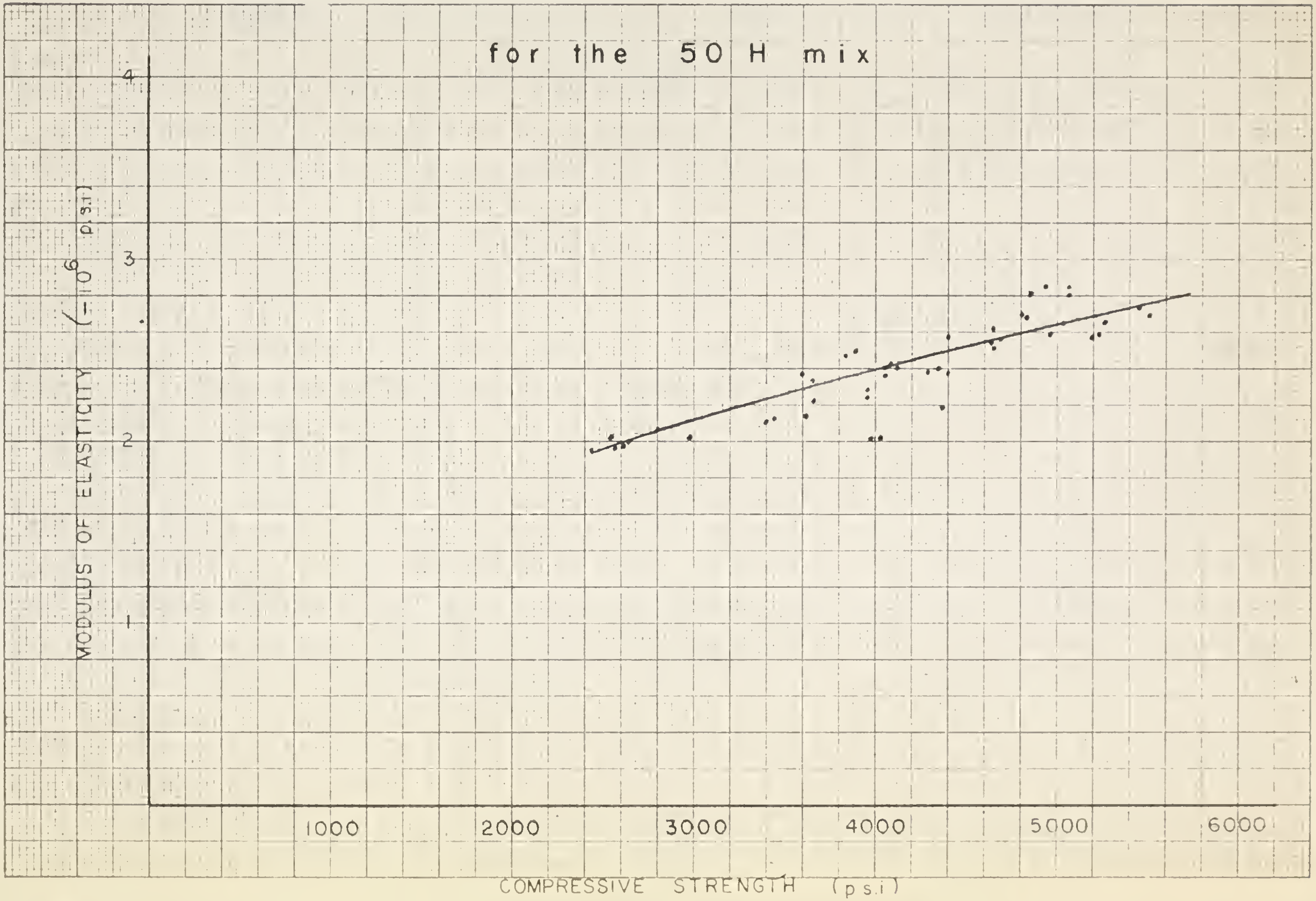
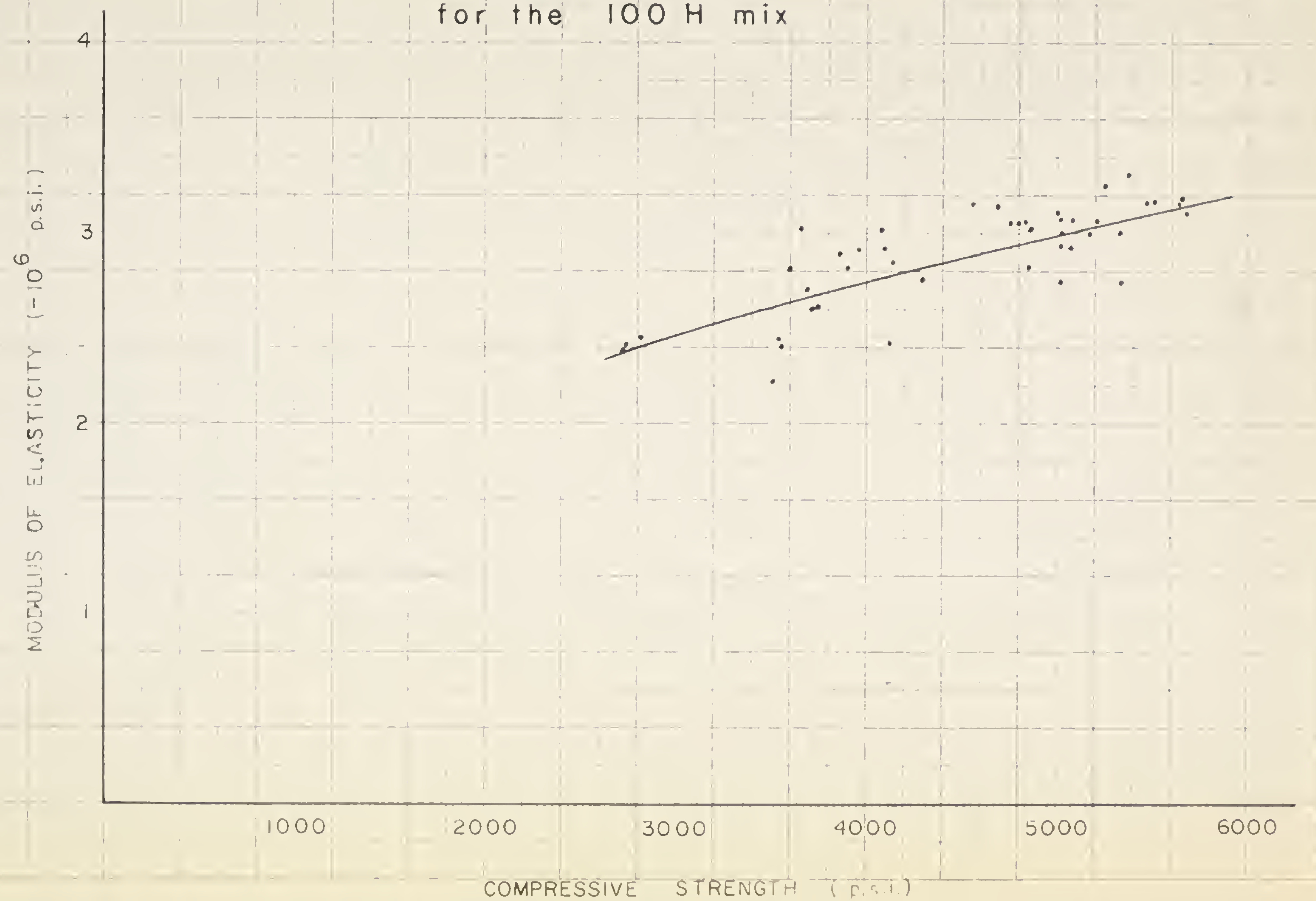


FIGURE 9

MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

for the 100 H mix



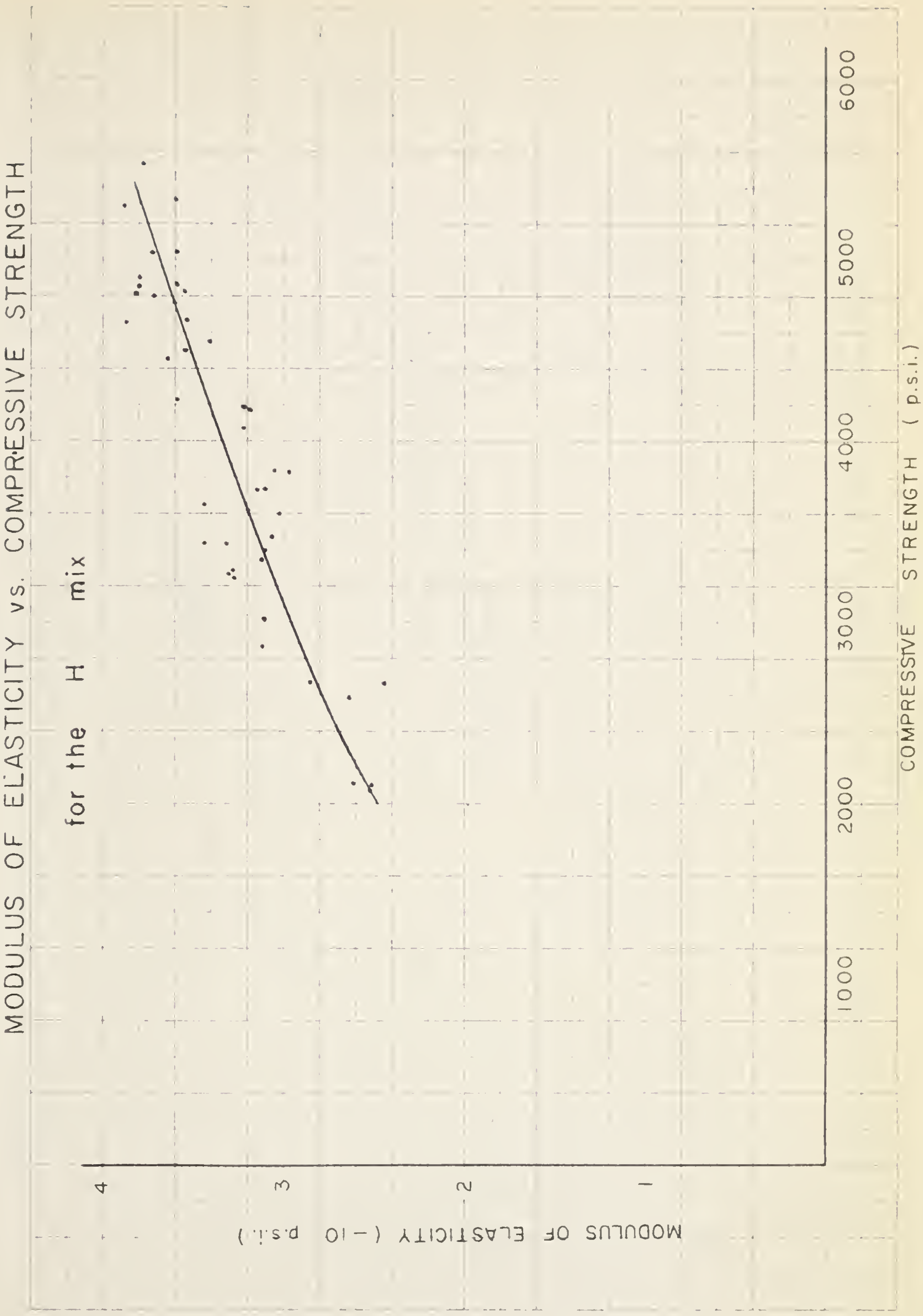


FIGURE 10

MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

for the H mix

FIGURE 11

SUMMARY OF THE MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH

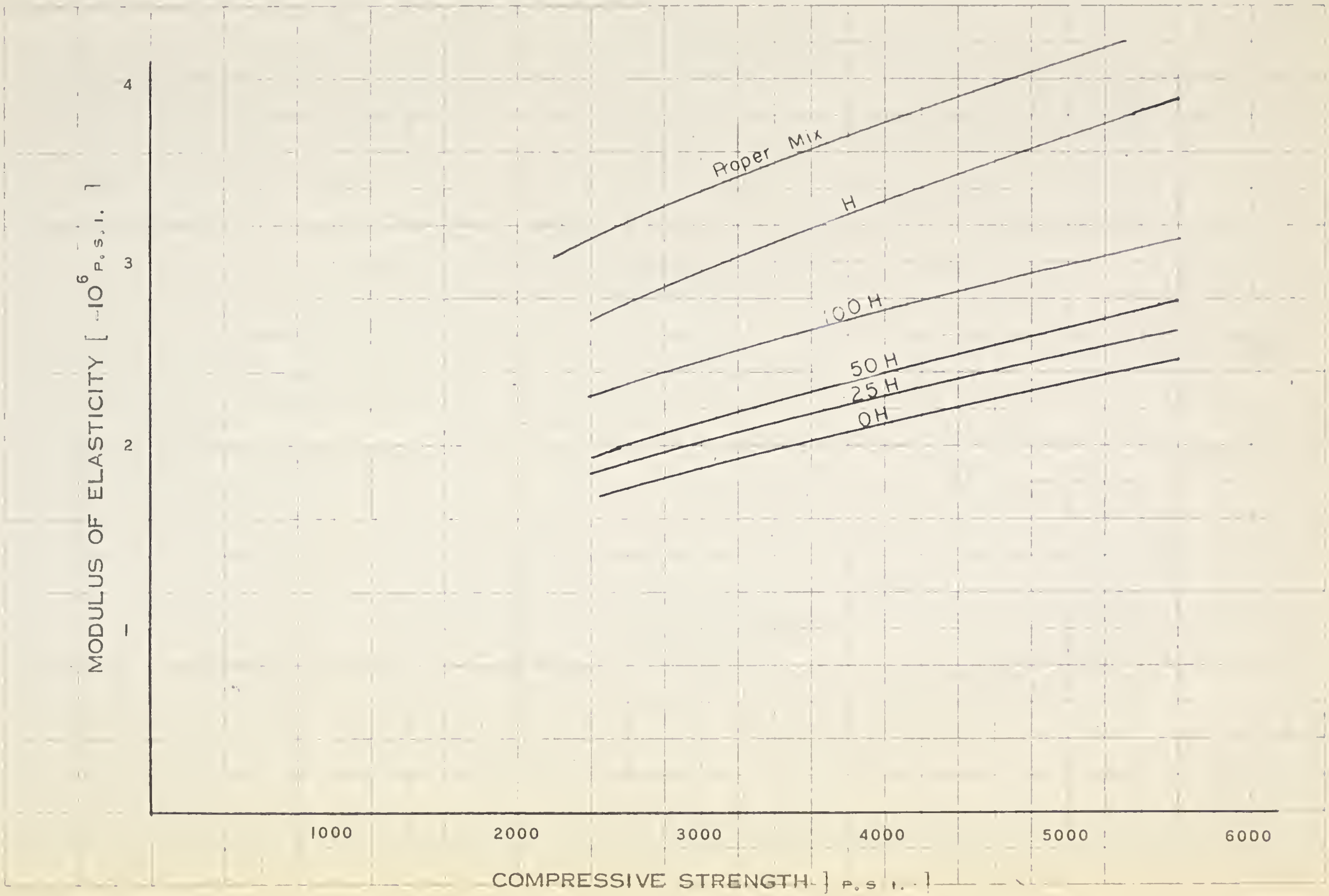
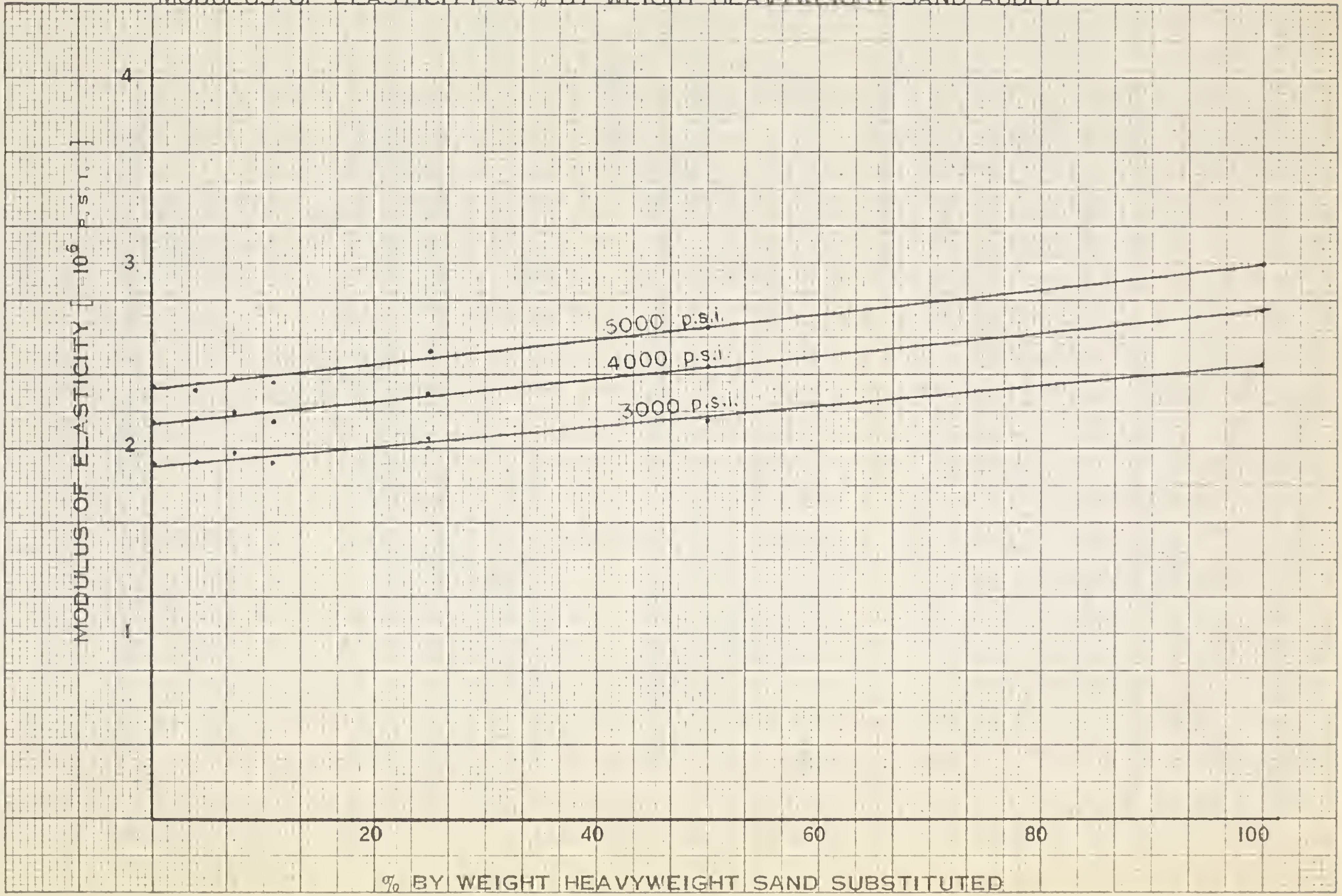


FIGURE 12

MODULUS OF ELASTICITY vs % BY WEIGHT HEAVYWEIGHT SAND ADDED



As can be seen from Figure 11 the modulus of elasticity for all lightweight concrete is approximately 65% of that of sand and gravel concrete having an equal volume of mortar and approximately 55% of that of a properly proportioned sand and gravel concrete. The modulus of elasticity of a lightweight concrete using all heavyweight sand is approximately 85% of that of a sand and gravel concrete having an equal volume of mortar. Therefore it can be seen that the mortar, though it does effect the modulus of elasticity of concrete, by no means completely controls it.

For each combination of aggregate a plot was made of the modulus of elasticity versus the percentage by weight of heavyweight sand substituted at constant strengths of 3,000, 4,000 and 5,000 p.s.i. a straight line relationship was found between 0 and 100% sand substituted (Figure 12).

Since the relationship results in lines which are very nearly parallel at all these strengths a general equation was derived to show the effect on the modulus of elasticity by substitution of heavyweight sand at any strength between 3,000 and 5,000 p.s.i. It is:

$$E = E_L + 0.65 \frac{W}{100}$$

where

E = the modulus of elasticity of the concrete in question

E_L = the modulus of elasticity of concrete made from all lightweight aggregates at the same compressive strength

W = the percentage by weight of heavyweight sand substituted

The cylinders from the 42 day tests were weighed immediately after removal from the moist room. To enable an accurate comparison of unit weights, compensation was made for the variation in heights of cylinders. Many cylinder diameters were measured, but the variations found did not change the cross-sectional area more than ± 0.5%. The cylinders were therefore assumed to all have a diameter of 6". From these weights and dimensions the average unit weight of each concrete employing one combination of aggregates was determined (Table 5).

A dimensionless plot (Figure 13) was then made between the ratio of the modulus of elasticity over the modulus of elasticity of a sand and gravel concrete and the ratio of the unit weight over the unit weight of sand and gravel concrete for compressive strengths of 3,000, 4,000 and 5,000 p.s.i. The modulus of elasticity obtained by Richart and Jensen [2)] for a properly proportioned sand and gravel concrete was used in this investigation. Since oversanded concrete such as the H mix from this investigation is known to have a modulus of elasticity inferior to that of a properly proportioned concrete the results from this mix were not used.

The unit weight of the sand and gravel concrete from Richart and Jensen was not given for 42 day age. However, their 7 day unit weights averaged out to essentially the same value as was obtained

from the H mix and this value was used.

A straight line relationship was found to apply for

$$\frac{E}{E_{SG}} \quad \text{vs.} \quad \frac{WT}{WT_{SG}}$$

between 3,000 and 5,000 p.s.i. for any one strength. The variation in strength caused very little difference in the relationship (see Figure 13). However, There was a very slight tendency for higher strength concretes to be less effected by a reduction in unit weight.

Although Richart and Jensen have used the initial tangent modulus instead of the secant modulus used in this investigation the difference will be little since the stress strain curve for concrete is nearly a straight line up to the working stress.

TABLE 5

RELATIVE WEIGHTS

Age 42 Days

Mix		1	2	3	Total Weight	Ave. Weight	Total Height	Ave. Height	Corr.(12") Weight
25-H	W	28.6	28.6	28.7	85.9	28.63			28.6
	L	12-1/16	11-15/16	12			36	12	
30-H	W	29.1	29.0	29.2	87.3	29.10			28.7
	L	12-1/8	12-1/8	12-3/16			36-7/16	12.15	
35-H	W	29.4	29.1	29.3	87.8	29.27			28.8
	L	12-1/4	12-1/8	12-3/16			36-9/16	12.19	
40-H	W	28.6	29.4	29.1	87.1	29.03			28.9
	L	11-15/16	12-3/16	12-1/8			36-1/4	12.08	
50-H	W	29.3	29.1	29.2	87.6	29.20			28.8
	L	12-1/8	12-3/16	12-1/8			36-7/16	12.15	
								Average	28.8
25-OH	W	20.25	21.0	20.85	62.10	20.70			20.5
	L	12-1/16	12-1/8	12-1/8			36-5/16	12.10	
30-OH	W	21.25	21.15	21.2	63.60	21.20			20.9
	L	12-3/16	12-1/8	12-1/8			36-7/16	12.15	
35-OH	W	21.35	21.35	21.5	64.20	21.40			21.2
	L	12-3/16	12-1/8	12-1/8			36-7/16	12.15	
40-OH	W	21.5	21.4	21.4	64.3	21.43			21.2
	L	12-1/8	12-1/8	12-1/8			36-3/8	12.125	
50-OH	W	21.4	21.6	21.2	64.2	21.40			21.2
	L	12-1/8	12-3/16	12-1/8			36-7/16	12.15	
								Average	21.0

[illegible]

TABLE 5 (CONT'D.)

Age 42 Days

Mix		1	2	3	Total Weight	Ave. Weight	Total Height	Ave. Height	Corr.(12") Weight
25-100H	W	24.0	23.4	23.55	70.95	23.65			23.5
	L	12-3/16	11-15/16	12-1/16			36-3/16	12.06	
30-100H	W	24.0	23.6	23.75	71.35	23.78			23.6
	L	12-3/16	11-15/16	12			36-1/8	12.04	
35-100H	W	24.0	23.95	24.0	71.95	23.98			23.7
	L	12-1/8	12-1/8	12-1/8			36-3/8	12.125	
40-100H	W	24.25	24.1	24.5	72.85	24.28			24.0
	L	12-3/16	12-1/8	12-3/16			36-1/2	12.167	
50-100H	W	24.0	24.25	24.3	72.55	24.18			23.8
	L	12-1/8	12-3/16	12-3/16			36-1/2	12.167	
								Average	23.7

Mix		1	2	3	Total Weight	Ave. Weight	Total Height	Ave. Height	Corr.(12") Weight
25-50H	W	22.3	22.75	22.45	67.5	22.5			22.1
	L	12-3/16	12-1/4	12-1/4			36-11/16	12.23	
30-50H	W	22.3	22.1	21.9	66.3	22.1			21.8
	L	12-3/16	12-1/8	12-1/8			36-7/16	12.15	
35-50H	W	22.55	22.15	22.3	67.0	22.33			22.2
	L	12-1/8	12	12-1/16			36-3/16	12.06	
40-50H	W	22.5	22.45	22.7	67.65	22.55			22.2
	L	12-1/8	12-1/8	12-1/4			36-1/2	12.167	
50-50H	W	22.5	22.9	22.8	68.20	22.73			22.4
	L	12-1/16	12-3/16	12-3/16			36-7/16	12.15	
								Average	22.2

TABLE 5 (CONT'D.)

Age 42 Days

Mix		1	2	3	Total Weight	Ave. Weight	Total Height	Ave. Height	Corr.(12") Weight
25-25H	W	21.45	21.45	21.45	64.35	21.45			21.4
	L	12	12-1/16	12-1/8			36-3/16	12.06	
30-25H	W	20.95	21.25	20.9	63.10	21.03			20.9
	L	12-1/16	12-3/16	12			36-1/4	12.08	
35-25H	W	21.65	21.65	21.6	64.90	21.63			21.4
	L	12-1/8	12-1/8	12-1/8			36-6/16	12.15	
40-25H	W	21.75	21.95	21.9	65.60	21.87			21.5
	L	12-1/8	12-3/16	12-1/4			36-9/16	12.19	
50-25H	W	22.1	22.15	22.3	66.55	22.18			21.9
	L	12-1/8	12-1/8	12-1/8			36-6/16	12.125	
								Average	21.4
25-10.7H	W	21.05	20.9	20.9	62.85	20.62			20.4
	L	12-5/32	12-3/16	12-1/8			36-15/32	12.16	
30-10.7H	W	21.1	21.4	21.1	63.6	21.2			21.0
	L	12-1/8	12-5/32	12-3/32			36-3/8	12.125	
35-10.7H	W	21.2	21.1	21.0	63.3	21.1			21.0
	L	12-1/8	12	12			36-1/8	12.04	
40-10.7H	W	21.5	21.5	21.5	64.5	21.5			21.2
	L	12-1/16	12-3/16	12-1/8			36-3/8	12.125	
50-10.7H	W	21.6	21.4	21.6	64.6	21.5			21.4
	L	12-1/8	12	12-1/16			36-3/16	12.06	
								Average	21.0

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[illegible]

TABLE 5 (CONT'D.)

Age 42 Days

Mix		1	2	3	Total Weight	Ave. Weight	Total Height	Ave. Height	Corr.(12") Weight
25-7.4H	W	20.6	20.7	20.5	61.8	20.6			20.4
	L	12	12-1/16	12-1/4			36-5/16	12.10	
30-7.4H	W	20.9	20.75	20.75	62.4	20.8			20.8
	L	12-1/32	12	12			36-1/32	12.02	
35-7.4H	W	20.65	20.35	21.0	62.0	20.65			20.2
	L	12-1/8	12-7/16	12-1/4			36-13/16	12.27	
40-7.4H	W	20.95	20.9	20.8	62.65	20.88			20.9
	L	12	12	12			36	12	
50-7.4H	W	21.0	20.7	20.85	62.55	20.85			20.6
	L	12-3/16	12-1/8	12-1/16			36-3/8	12.13	
								Average	20.6

Mix		1	2	3	Total Weight	Ave. Weight	Total Height	Ave. Height	Corr.(12") Weight
25-4H	W	19.95	20.15	21.6	61.7	20.57			20.6
	L	12	11-15/16	12			35-15/16	11.98	
30-4H	W	20.55	20.7	20.5	61.85	20.62			20.6
	L	12-1/16	12	11-15/16			36	12	
35-4H	W	20.8	20.8	20.9	62.5	20.83			20.6
	L	12-3/16	12-1/16	12-1/8			36-3/8	12.13	
40-4H	W	21.3	21.2	20.95	63.45	21.15			20.8
	L	12-3/16	12-1/4	12-1/8			36-9/16	12.19	
50-4H	W	21.2	21.5	21.4	64.1	21.37			21.1
	L	12-1/8	12-3/16	12-1/8			36-7/16	12.15	
								Average	20.7

[illegible]

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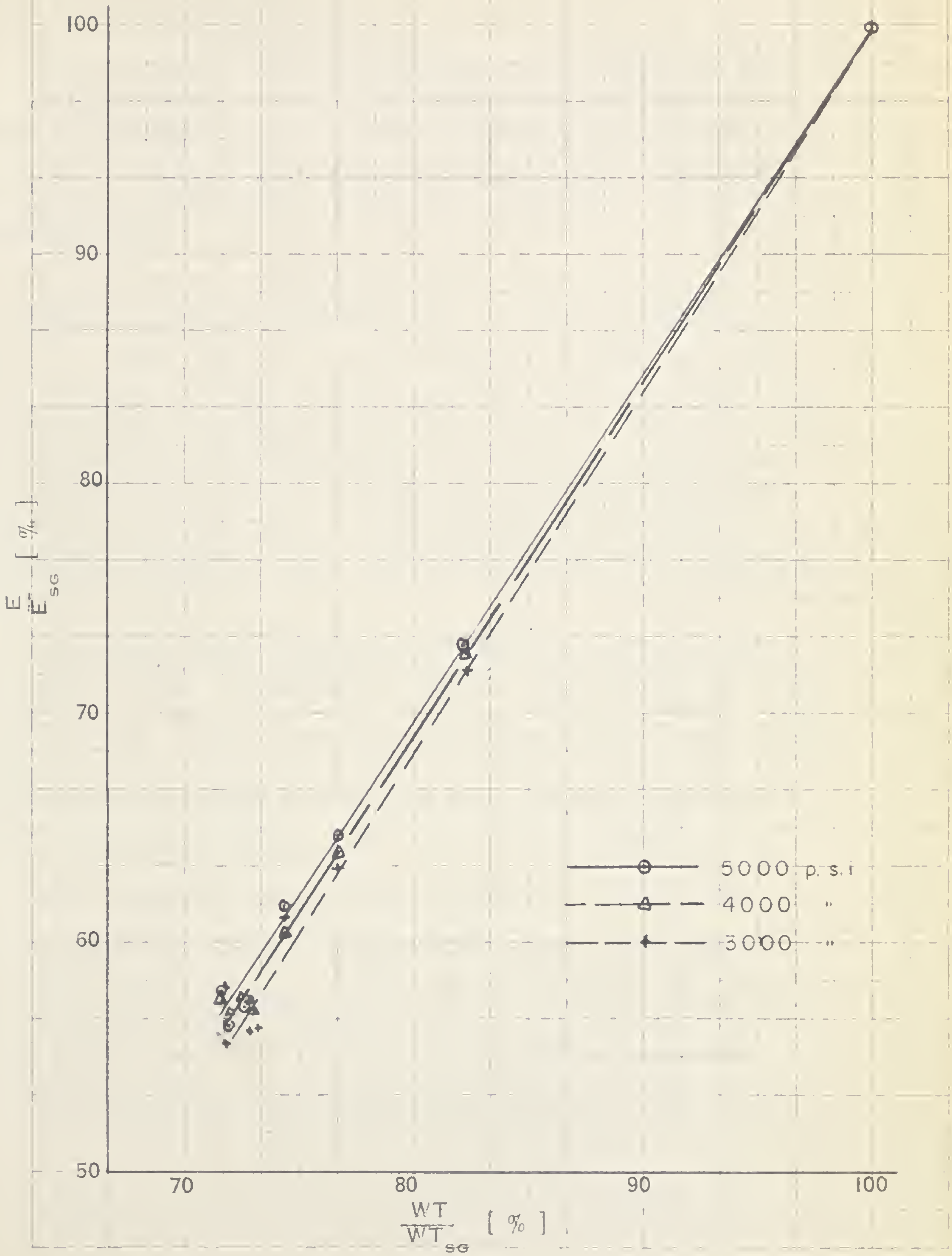
FIGURE 15

$\frac{E}{E_{SG}}$ vs. $\frac{WT.}{WT_{SG}}$

THE EFFECT OF WEIGHT ON THE MODULUS OF ELASTICITY



CHAPTER 6

COMPARISONS WITH PUBLISHED DATA

There is little information available on the Modulus of Elasticity of lightweight concrete, while information on the modulus of elasticity for lightweight concrete with heavyweight sand added is practically nil.

The best comparison available is in the report by Richart and Jensen [2)]. Tests were run on sand and gravel concrete, all lightweight concrete and lightweight concrete with 100% heavyweight sand. The relationships between the modulus of elasticity and the compressive strength are shown in comparison with the results from similar mixes obtained by this investigation and others in Figure 14.

The initial tangent modulus of elasticity was used by Richart and Jensen in their investigation. However, since the stress strain relationship for concrete is nearly a straight line up to the working stress the error induced by comparing it to the secant modulus is small. Richart and Jensen state: " ----- for deformations up to 50% of the ultimate deformation the relations of secant moduli of Haydite and gravel concretes are essentially the same as those of the initial moduli" . The materials used are expanded shales from entirely different sources ~~as~~ than those used in this investigation. This could explain the discrepancy that does occur. Other factors

which might contribute to the difference are: 1) their mixes varied from 1:4 to 1:5; 2) their percentages of fine aggregate to coarse aggregate varied from 37.5 to 62.5; and 3) their slumps varied from 1 inch to 10 inches.

As stated before the sand and gravel mix used in this investigation employed a mortar volume equal to that used in the lightweight mixes resulting in an extremely oversanded concrete (and a lower modulus of elasticity versus compressive strength relationship).

The results obtained by Simmonds [3)] give a comparison for the all lightweight aggregate concrete. His tests were run on aggregate produced in the same plant as those used in this investigation. The secant moduli at 0.45 f'_c was used. The relationship between the modulus of elasticity and compressive strength is shown in Figure 14. The curve obtained by Simmonds is approximately 5% lower than the results obtained from this investigation. This is probably due to the difference in materials produced four years apart and the difference in mixing techniques. Simmonds' aggregates were pre-soaked and the mixing was done by hand.

The average unit weight of the OH mixes (all lightweight concrete) was 107 lbs. per cubic foot. This is approximately 5% higher than is usually obtained for this material. Using a straight line

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3) S. H. Simmonds - The Stress Strain Relationship for Lightweight Concrete - Masters Thesis - University of Alberta, 1956

relationship obtained in Figure 13 for $\frac{E}{E_{S\&G}}$ vs. $\frac{WT}{WT_{S\&G}}$ a unit weight of 105 lbs. per cubic foot will give a modulus of elasticity quite close to that of Simmonds'. However, no unit weights were included in Simmonds' thesis so this could not be verified.

Some results obtained by Shideler [4)] are also recorded in Figure 14. The curve represents the results from an expanded shale and an expanded slate. The difference between the two relationships was practically negligible, therefore an average curve is shown. Shideler used the secant modulus at 0.3 f'_c . However, he stated: " --- almost identical values were obtained at 0.45 f'_c " . The unit weights of the material were given, but the relationship of $\frac{E}{E_{SG}}$ vs. $\frac{WT}{WT_{SG}}$ for this material plot considerably away from the results obtained in this investigation. The unit weight of Shideler's concrete was approximately 90 lbs. per cubic foot. It therefore seems obvious that the relationship developed in this investigation holds only for the aggregate tested. However, it is felt that some similar straight line relationship of $\frac{E}{E_{SG}}$ vs. $\frac{WT}{WT_{SG}}$ will hold for any one particular lightweight aggregate. One possible reason for this difference is that Shideler combined his aggregates in a dry state, while the aggregates used in this investigation were combined while having a moisture content in excess of 10%. This could incur an appreciable difference to the unit weight of the concrete.

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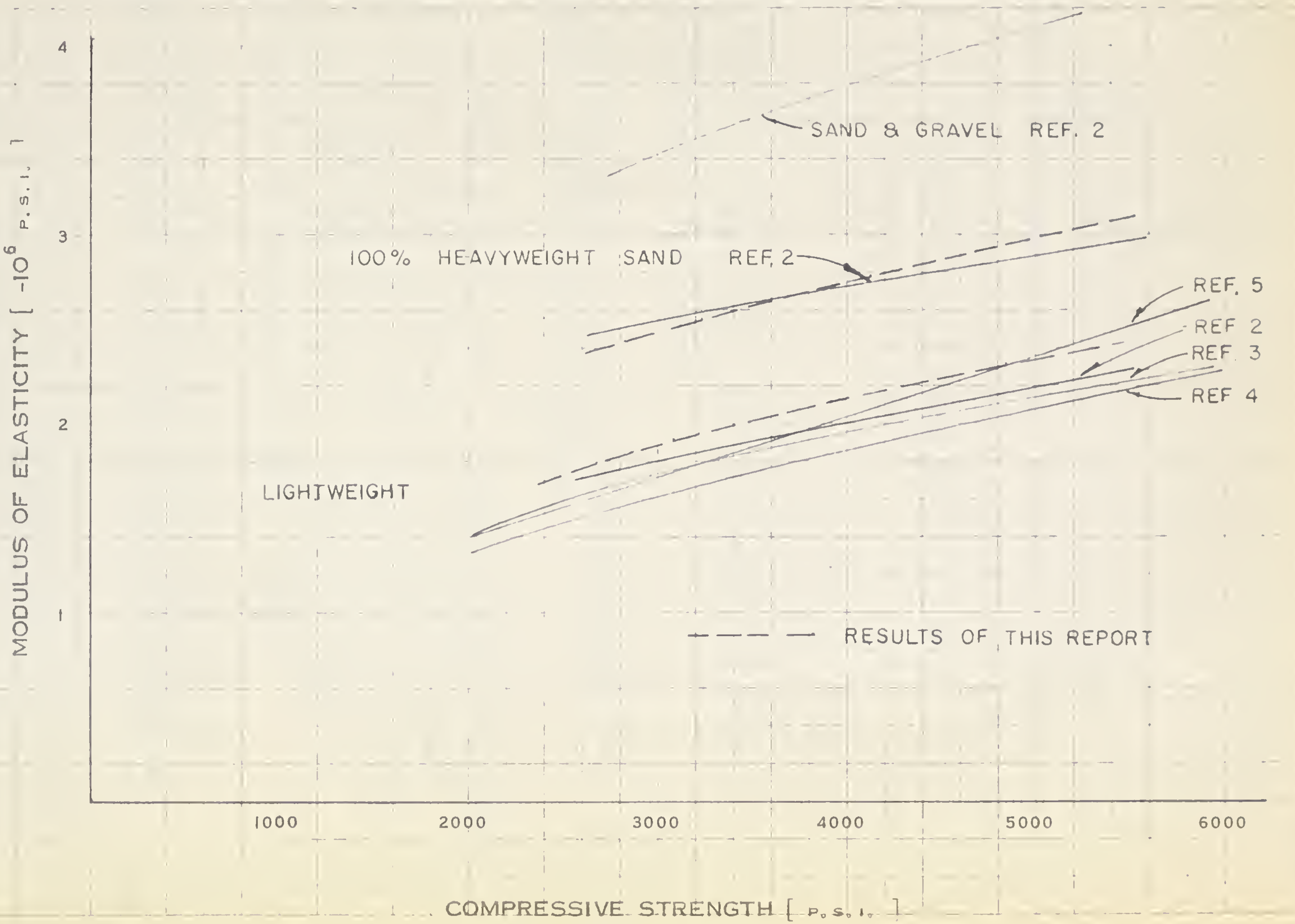
4) J. J. Shideler - Lightweight Aggregate Concrete for Structural Use. Proceedings of the A.C.I. Vol. 54 Oct 1957 p.p. 299-328

FIGURE 14

COMPARISON OF THE MODULUS OF ELASTICITY vs. COMPRESSIVE STRENGTH



It is interesting to note though the unit weight for appropriate concretes used by Richart and Jensen (aggregates combined in wet condition) is about 104 lbs. per cubic foot for all lightweight concrete. The $\frac{E}{E_{SG}}$ vs. $\frac{WT}{WT_{SG}}$ relationship is quite close to that obtained in this investigation as is shown in Figure 15. The results from Richart and Jensen's lightweight concrete with 100% heavyweight sand are also shown to fall in fair agreement with the established relationship. In the latter case the aggregates were combined dry to the concrete. However, this will have a smaller effect on the results than in the case of all lightweight concrete since the heavyweight sand absorption is not appreciable and the higher unit weight is not subjected to as much percentage change by the absorption.

In general Richart and Jensen's results tend to support the relationships obtained in this investigation.

G. H. Nelsen and Otto C. Frei [5)] obtained a straight line relationship between the modulus of elasticity and the compressive strength for all lightweight concrete made from an expanded shale. It is shown in comparison with others in Figure 14. It is somewhat higher than that obtained in this investigation at higher strengths.

In summary the data obtained is within the range of results available for comparison, which in general would seem to verify the

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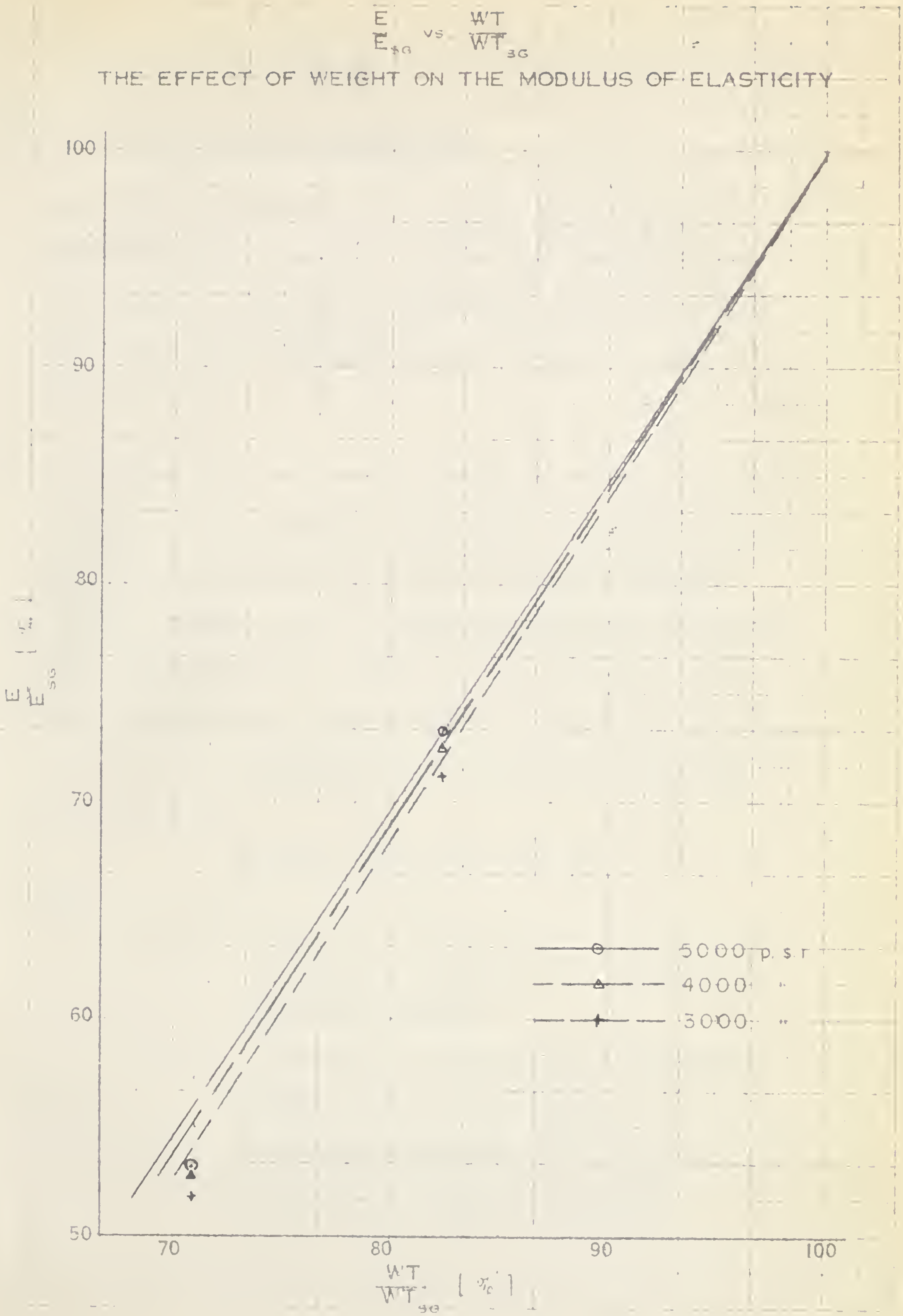
5) G. H. Nelsen and Otto C. Frei - Lightweight Concrete Proportioning and Control - Proceedings of the A.C.I. Vol. 54 Jan '58 p.p. 605-621

author's conclusions. However, little if no information is available on the modulus of elasticity other than for heavyweight and all lightweight concretes. Because of the great variation in quality of lightweight aggregate a strict comparison is impossible.

FIGURE



E / E_SG vs WT / WT_SG
THE EFFECT OF WEIGHT ON THE MODULUS OF ELASTICITY
100
90
80
70
60
50
70
80
90
100
WT / WT_SG (%)
E / E_SG
5000 p.s.i
4000 "
3000 "

CHAPTER 7

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

1. The modulus of elasticity of an expanded shale concrete is approximately 55% of that of sand and gravel concrete while the modulus of elasticity of lightweight concrete using 100% heavyweight sand is approximately 75% of that of sand and gravel concrete.

2. For constant mortar volumes the modulus of elasticity of lightweight concrete is linearly proportional to the percent heavyweight sand substituted into the mix at any one strength. The total increase from 0 to 100% heavyweight sand substituted being approximately 0.6 x 10^6 p.s.i. at any strength between 3,000 and 5,000 p.s.i. The modulus of elasticity of lightweight concrete with any heavyweight sand content can be described by the equation:

$$E = E_L + \frac{W}{100} \times 0.65 \times 10^6 \text{ psi}$$

where

E = the modulus of elasticity of the lightweight concrete in question

E_L = the modulus of elasticity of all lightweight concrete at the same strength

W = the percent heavyweight sand substituted into the lightweight concrete in question

3. If heavyweight sand is substituted for lightweight fines the modulus of elasticity increases in direct proportion to the resulting increase in unit weight. This was found to hold true for all percentages substituted including sand and gravel concrete for any strength between 3,000 and 5,000 p.s.i. In this strength range the substitution of up to 12% heavyweight sand is of little value in increasing the modulus of elasticity. The substitution of 100% heavyweight sand for lightweight fines in lightweight concrete raises the modulus of elasticity from 56% to approximately 73% of that for normal sand and gravel concrete. In gaining this increase in the modulus of elasticity the unit weight is raised from 72% to approximately 82% of that of normal sand and gravel concrete.

4. The mortar (cement, sand and water) does not entirely control the modulus of elasticity of concrete since complete substitution of heavyweight mortar in lightweight concrete results in an 55% increase of the difference between lightweight and sand and gravel concrete having equal mortar volumes.

5. The addition of as little as 7.4% heavyweight sand to lightweight concrete showed a considerable increase in the workability of the lightweight concrete. Because of this it would be possible to proportion lightweight concrete with a lower mortar volume.

6. The modulus of elasticity is considerably lower for an oversanded concrete than that of a properly proportioned concrete.

The following recommendations are made for subsequent investigations into the modulus of elasticity of lightweight concrete:

- A study to determine the ideal proportioning of lightweight aggregates for optimum strength and modulus of elasticity conditions.

- The proportioning used in this investigation yielded slightly harsh mixes at low strength for all lightweight concrete. Therefore, if possible, the mortar volumes should be larger at lower strength than at higher strength concrete.

- A study to determine the effect of age on the modulus of elasticity. There was some indication in this investigation that concrete at a younger age has a lower modulus of elasticity at any one compressive strength.

- A study to determine the effect on the modulus of elasticity due to a change in moisture content of the aggregate used.

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